

Lecture 9  
2018/2019

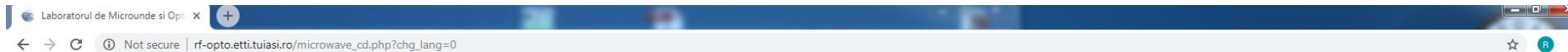
# Microwave Devices and Circuits for Radiocommunications

# 2018/2019

- 2C/1L, **MDCR**
- Attendance at minimum 7 sessions (course + laboratory)
- Lectures- **associate professor Radu Damian**
  - Friday 09-11, 11.13
  - E – 50% final grade
  - problems + (2p atten. lect.) + (**3 tests**) + (bonus activity)
    - 3p=+0.5p
  - all materials/equipments authorized
- Laboratory – **associate professor Radu Damian**
  - Wednesday 12-14, 11.12 odd weeks
  - L – 25% final grade
  - P – 25% final grade

# Materials

■ <http://rf-opto.etti.tuiasi.ro>



Main **Courses** Master Staff Research Students Admin

**Microwave CD** Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

## Microwave Devices and Circuits for Radiocommunications (English)

### Course: MDCR (2017-2018)

**Course Coordinator:** Assoc.P. Dr. Radu-Florin Damian  
**Code:** EDOS412T  
**Discipline Type:** DOS; Alternative, Specialty  
**Credits:** 4  
**Enrollment Year:** 4, Sem. 7

### Activities

**Course:** Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:  
**Laboratory:** Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

### Evaluation

Type: **Examen**

**A:** 50%, (Test/Colloquium)  
**B:** 25%, (Seminary/Laboratory/Project Activity)  
**D:** 25%, (Homework/Specialty papers)

### Grades

[Aggregate Results](#)

### Attendance

[Course](#)  
[Laboratory](#)

### Lists

[Bonus-uri acumulate \(final\)](#)  
[Studenti care nu pot intra in examen](#)

### Materials

#### Course Slides

[MDCR Lecture 1](#) (pdf, 5.43 MB, en, [ps](#))  
[MDCR Lecture 2](#) (pdf, 3.67 MB, en, [ps](#))  
[MDCR Lecture 3](#) (pdf, 4.76 MB, en, [ps](#))  
[MDCR Lecture 4](#) (pdf, 5.58 MB, en, [ps](#))

# Examen: Logarithmic scales

$$\text{dB} = 10 \cdot \log_{10} (P_2 / P_1)$$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

$$\text{dBm} = 10 \cdot \log_{10} (P / 1 \text{ mW})$$

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 $\mu$ W
-30 dBm	= 1 $\mu$ W
-60 dBm	= 1 nW

$$[\text{dBm}] + [\text{dB}] = [\text{dBm}]$$

$$[\text{dBm/Hz}] + [\text{dB}] = [\text{dBm/Hz}]$$

$$[x] + [\text{dB}] = [x]$$

# Examen

- Complex numbers arithmetic!!!!
- $z = a + j \cdot b ; j^2 = -1$

# Impedance Matching

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# Matching , from the point of view of power transmission

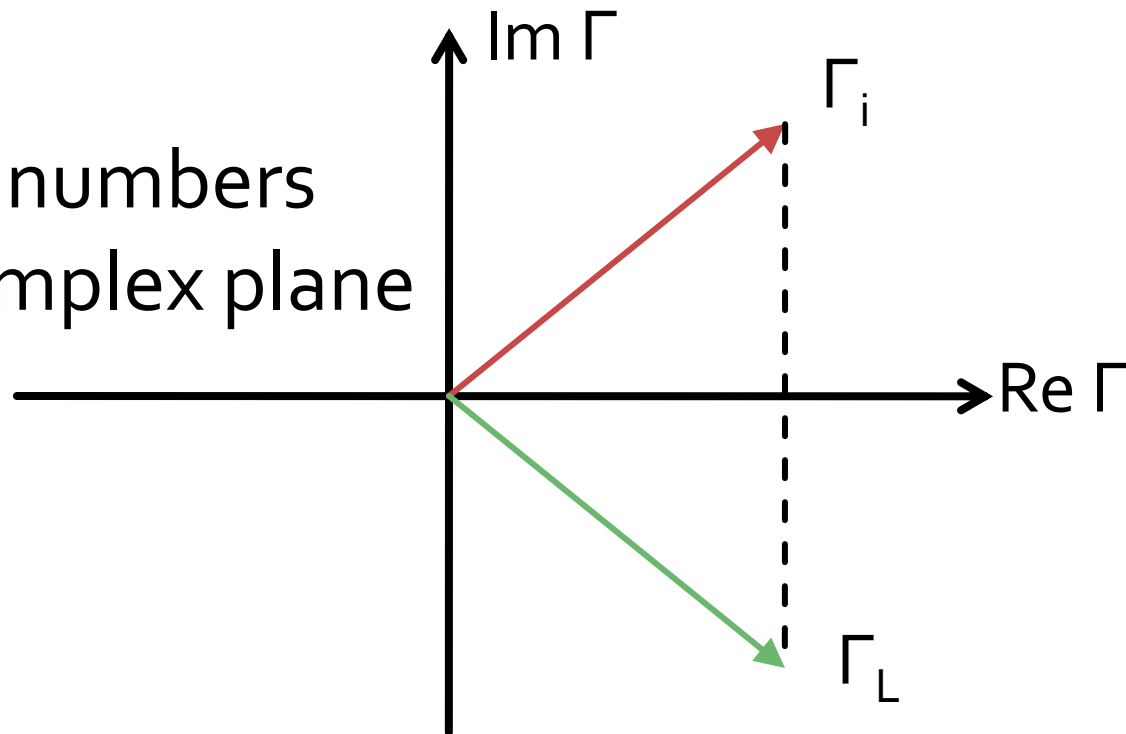
$$Z_L = Z_i^*$$

If we choose a real  $Z_0$

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- complex numbers
- in the complex plane



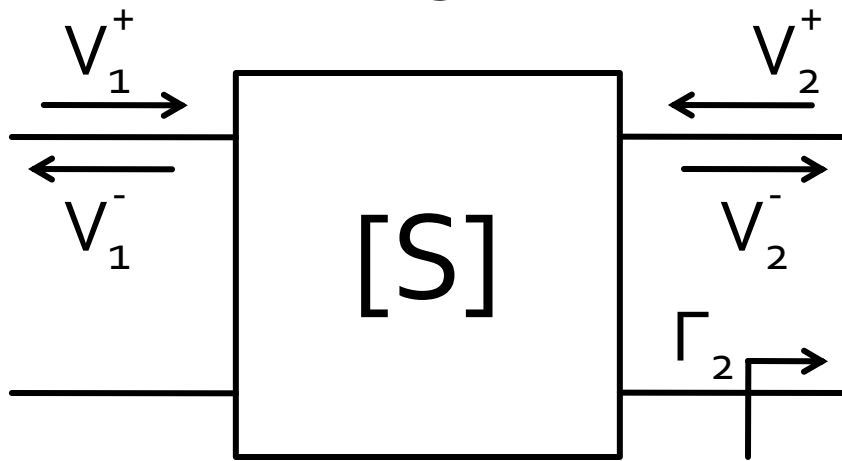
Lecture 3-4

# Microwave Network Analysis



# Scattering matrix – S

- Scattering parameters



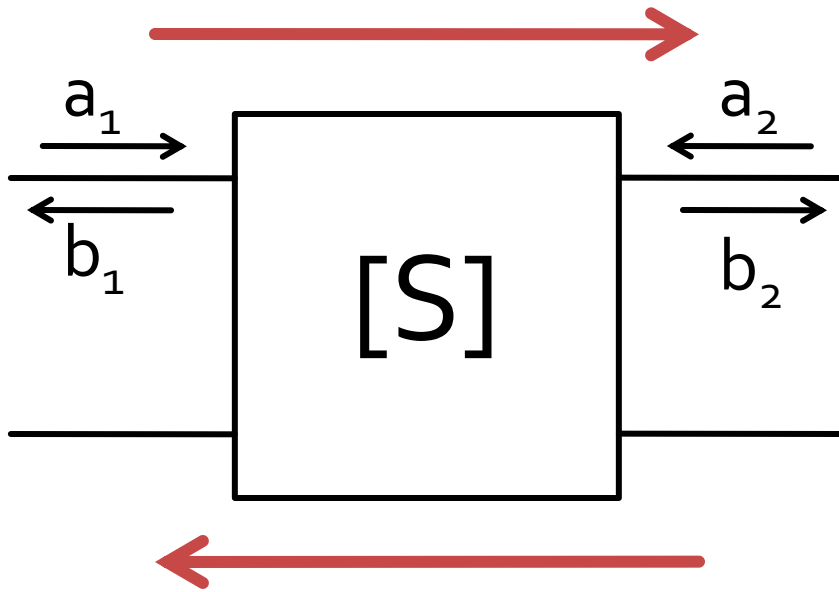
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0} \quad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0}$$

- $V_2^+ = 0$  meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

# Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{\text{Power in } Z_0 \text{ load}}{\text{Power from } Z_0 \text{ source}}$$

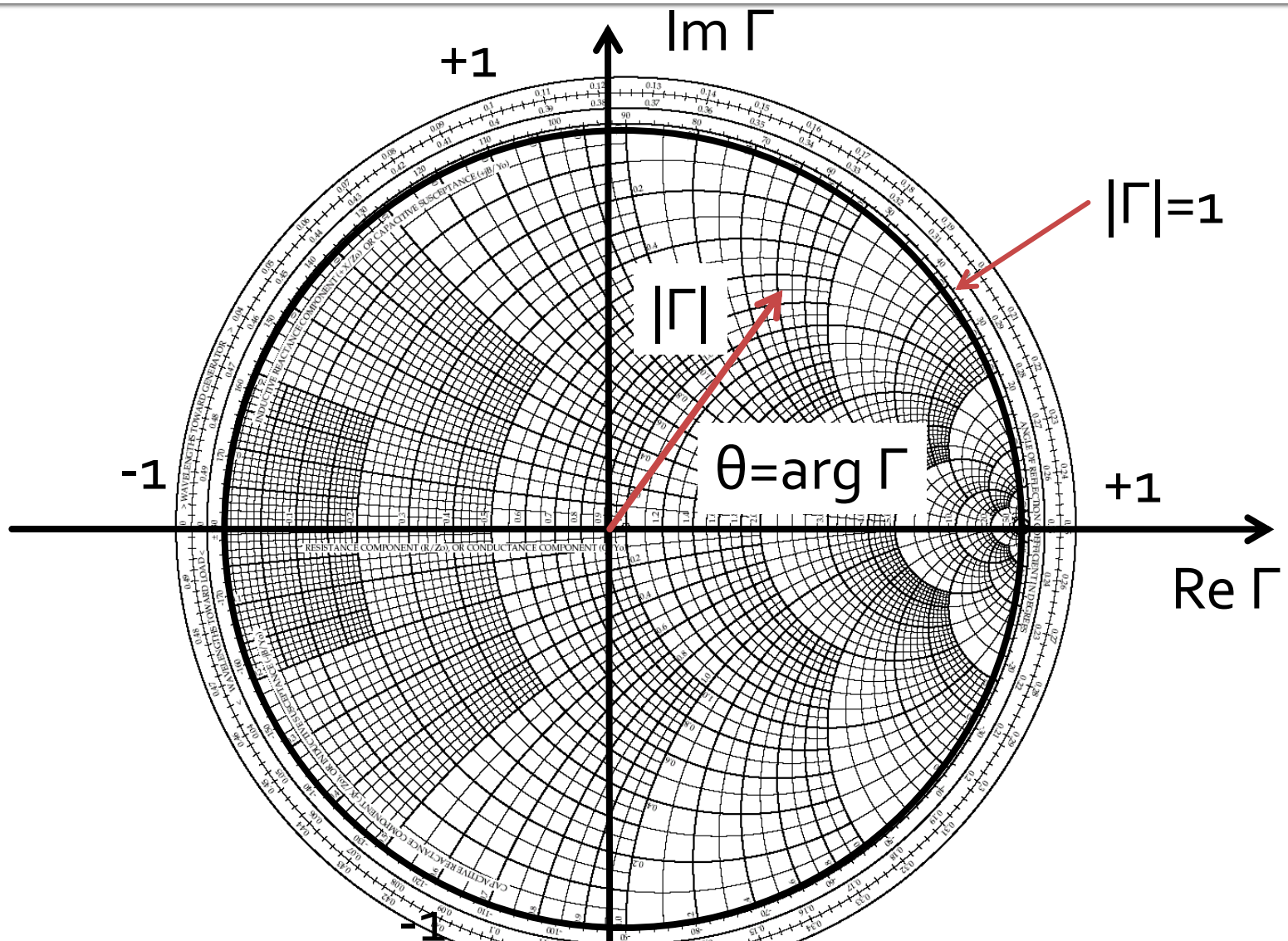
- $a, b$ 
  - information about signal power **AND** signal phase
- $S_{ij}$ 
  - network effect (gain) over signal power **including** phase information

Impedance Matching

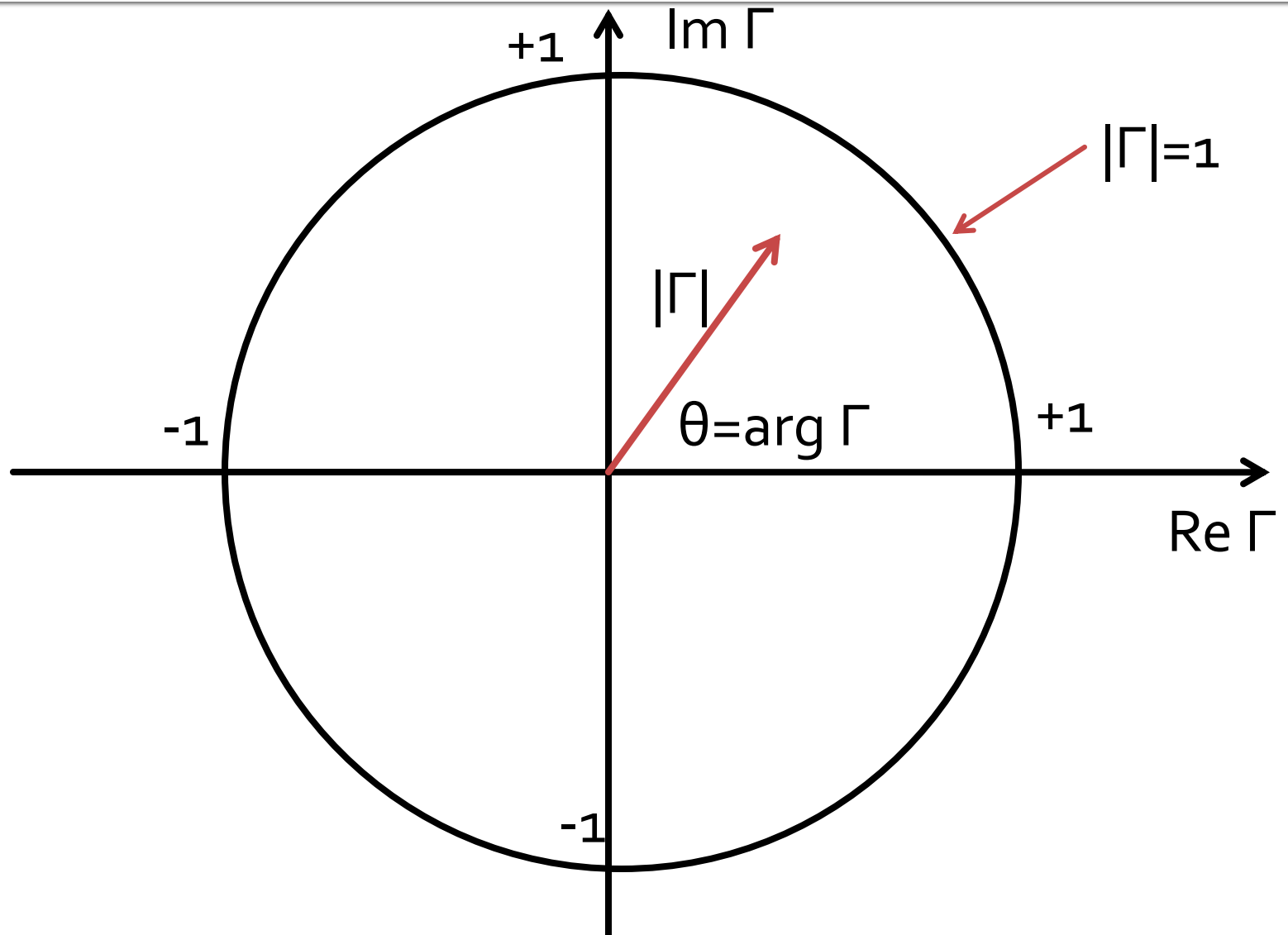
# The Smith Chart

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# The Smith Chart



# The Smith Chart



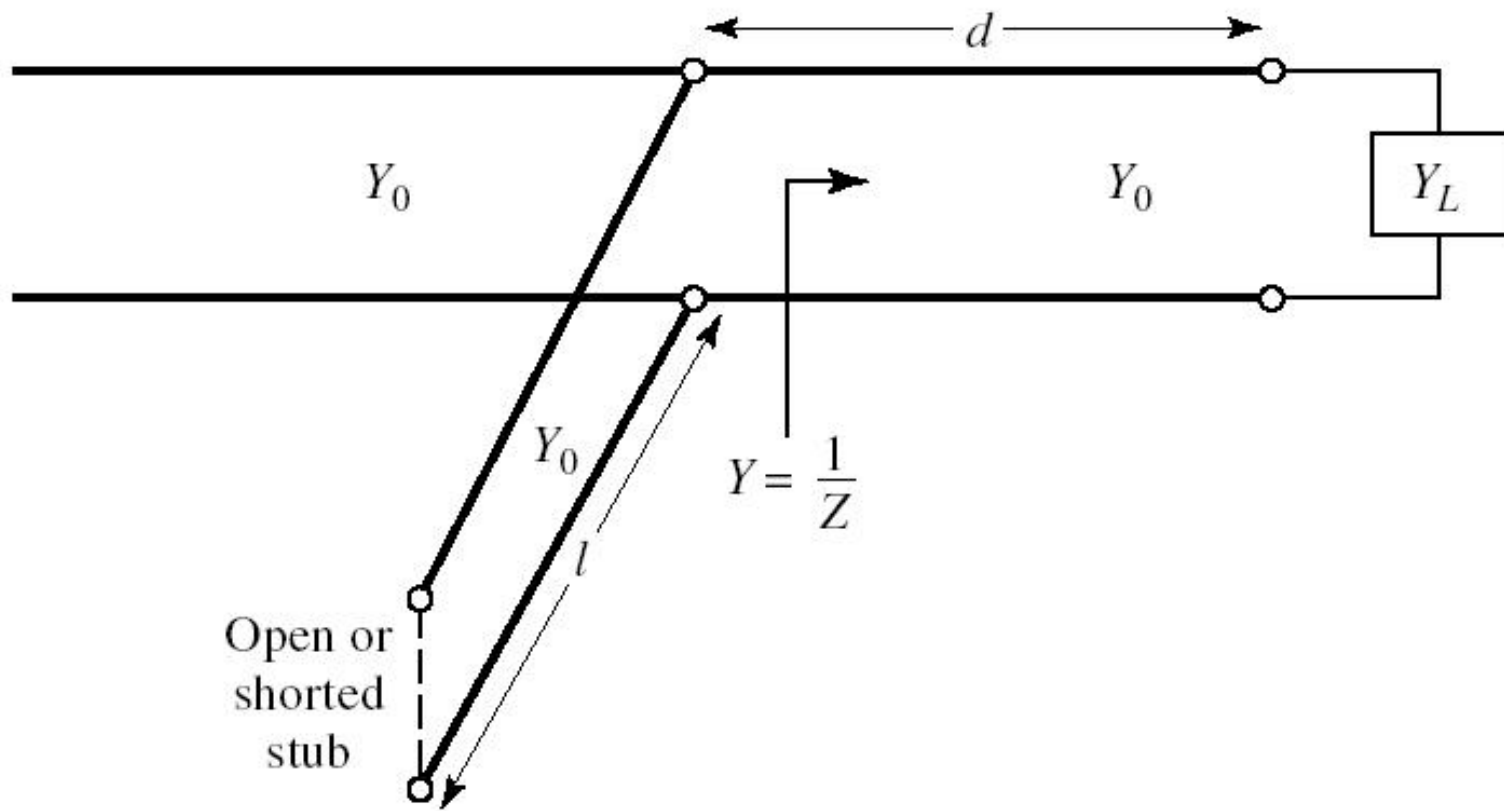
Impedance Matching with Stubs

# Impedance Matching

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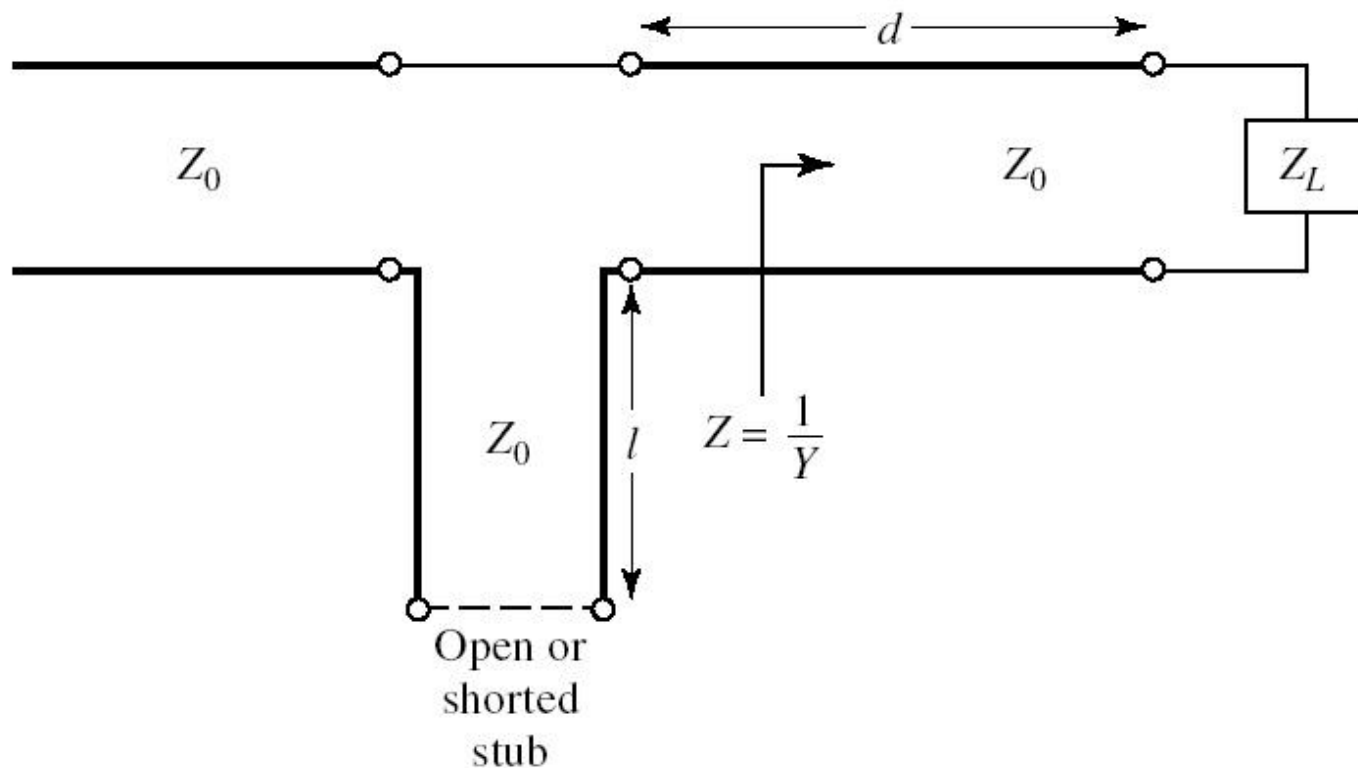
# Single stub tuning

- Shunt Stub



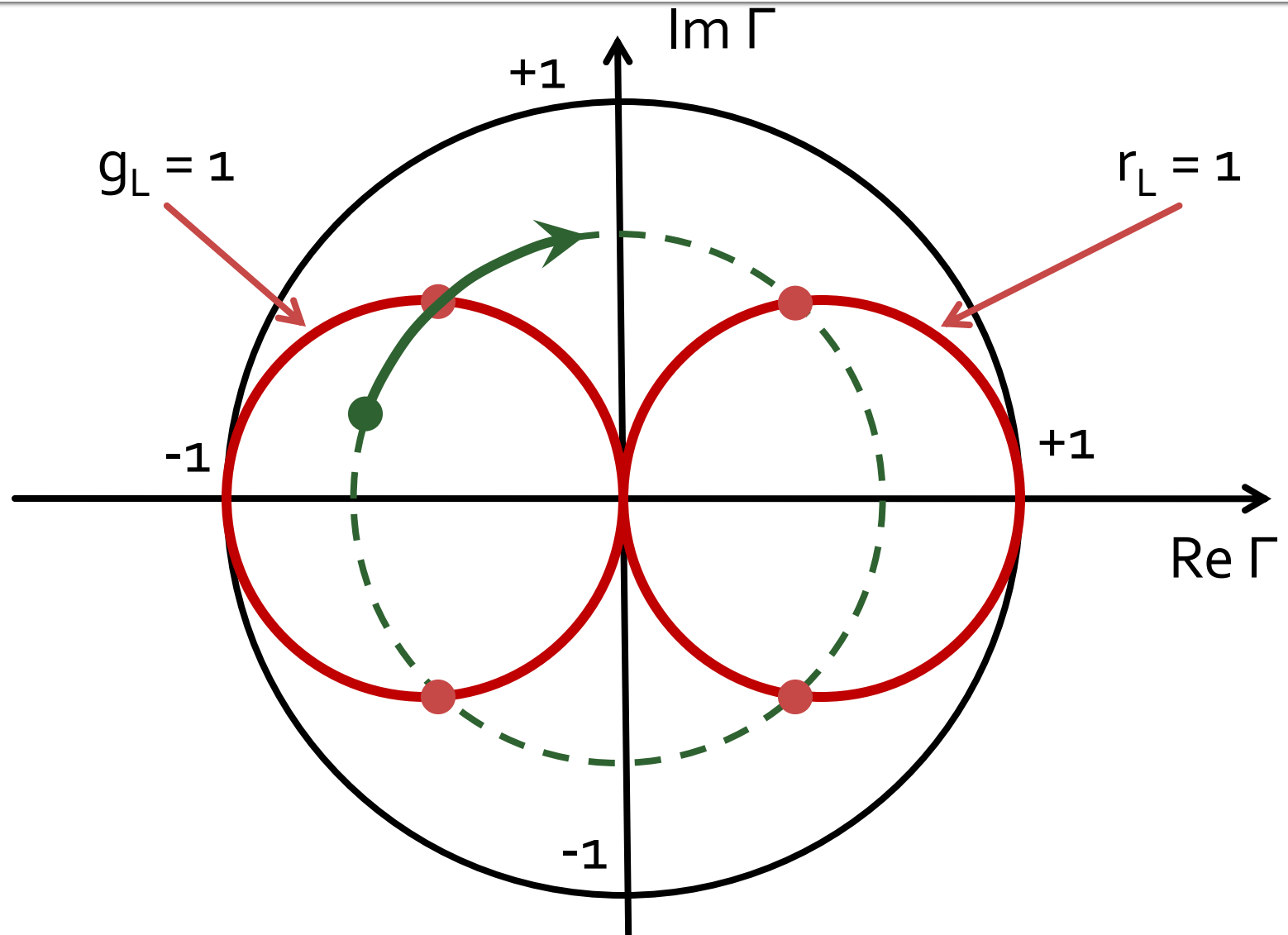
# Single stub tuning

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)





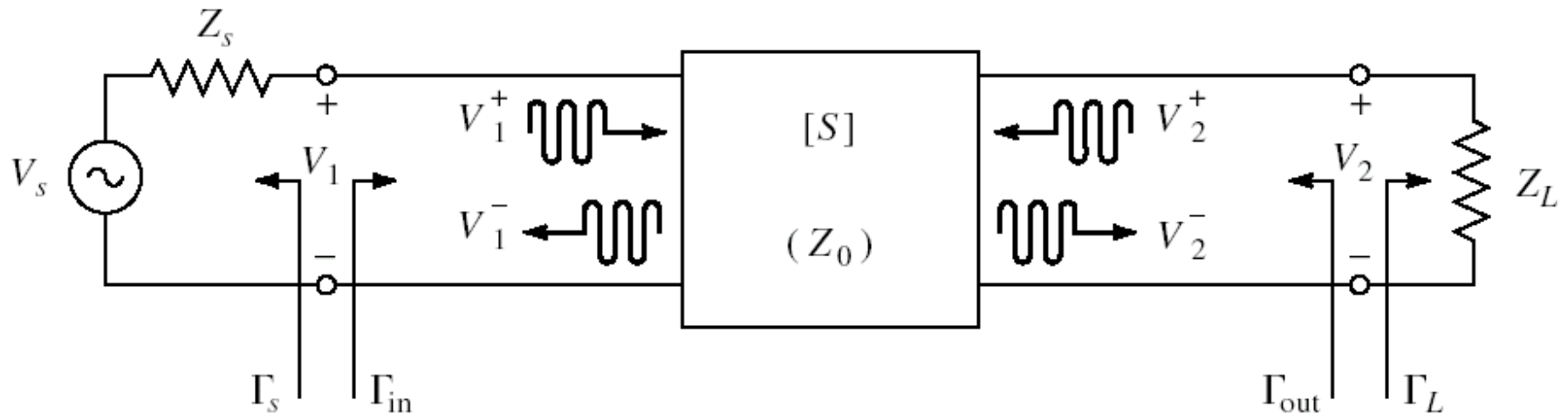
# Smith chart, $r=1$ and $g=1$



# Microwave Amplifiers

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# Amplifier as two-port



- Charaterized with S parameters
- normalized at  $Z_0$  (implicit  $50\Omega$ )
- Datasheets: S parameters for specific bias conditions

# Datasheets

## NE46100

VCE = 5 V, Ic = 50 mA

FREQUENCY (MHz)	S11		S21		S12		S22		K	MAG <sup>2</sup> (dB)
	MAG	ANG	MAG	ANG	MAG	ANG	MAG	ANG		
100	0.778	-137	26.776	114	0.028	30	0.555	-102	0.16	29.8
200	0.815	-159	14.407	100	0.035	29	0.434	-135	0.36	26.2
500	0.826	-177	5.855	84	0.040	38	0.400	-162	0.75	21.7
800	0.827	176	3.682	76	0.052	43	0.402	-169	0.91	18.5
1000	0.826	173	2.963	71	0.058	47	0.405	-172	1.02	16.3
1200	0.825	170	2.441	66	0.064	47	0.412	-174	1.08	14.0
1400	0.820	167	2.111	61	0.069	47	0.413	-176	1.17	12.4
1600	0.828	165	1.863	57	0.078	54	0.426	-177	1.15	11.4
1800	0.827	162	1.671	53	0.087	50	0.432	-178	1.14	10.6
2000	0.828	159	1.484	49	0.093	50	0.431	-180	1.17	9.5
2500	0.822	153	1.218	39	0.11	48	0.462	177	1.18	7.8
3000	0.818	148	1.010	30	0.135	46	0.490	174	1.16	6.3
3500	0.824	142	0.876	21	0.147	44	0.507	170	1.16	5.3
4000	0.812	137	0.762	13	0.168	38	0.535	167	1.14	4.3

VCE = 5 V, Ic = 100 mA

100	0.778	-144	27.669	111	0.027	35	0.523	-114	0.27	30.2
200	0.820	-164	14.559	97	0.029	29	0.445	-144	0.42	27.0
500	0.832	-179	5.885	84	0.035	38	0.435	-166	0.81	22.2
800	0.833	175	3.691	76	0.048	45	0.435	-173	0.95	18.8
1000	0.831	172	2.980	71	0.056	51	0.437	-176	1.05	16.0
1200	0.836	169	2.464	67	0.061	52	0.432	-178	1.11	14.0
1400	0.829	166	2.121	61	0.072	53	0.447	-180	1.12	12.6
1600	0.831	164	1.867	58	0.080	54	0.445	179	1.14	11.4

# S2P - Touchstone

- Touchstone file format (\*.s2p)

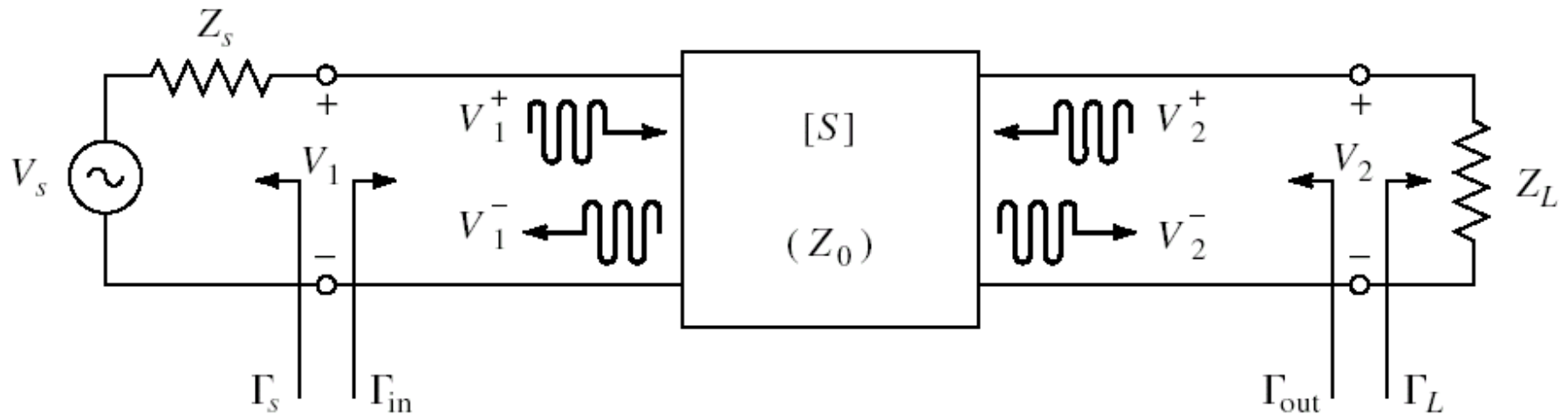
```
! SIEMENS Small Signal Semiconductors
! VDS = 3.5 V   ID = 15 mA
# GHz S MA R 50
! f      S11      S21      S12      S22
! GHz   MAG ANG   MAG ANG   MAG ANG   MAG ANG
1.000 0.9800 -18.0 2.230 157.0 0.0240 74.0 0.6900 -15.0
2.000 0.9500 -39.0 2.220 136.0 0.0450 57.0 0.6600 -30.0
3.000 0.8900 -64.0 2.210 110.0 0.0680 40.0 0.6100 -45.0
4.000 0.8200 -89.0 2.230 86.0 0.0850 23.0 0.5600 -62.0
5.000 0.7400 -115.0 2.190 61.0 0.0990 7.0 0.4900 -80.0
6.000 0.6500 -142.0 2.110 36.0 0.1070 -10.0 0.4100 -98.0
!
! f      Fmin  Gammaopt  rn/50
! GHz   dB   MAG ANG   -
2.000   1.00 0.72 27 0.84
4.000   1.40 0.64 61 0.58
```

Stability

# Microwave Amplifiers

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# Amplifier as two-port



- For an amplifier two-port we are interested in:
  - **stability**
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

# Stability

$$|\Gamma_{in}| < 1 \quad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- The limit between stability/instability

$$|\Gamma_{in}| = 1 \quad \log_{10} |\Gamma_{in}| = 0 \quad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| = 1$$

$$|S_{11} \cdot (1 - S_{22} \cdot \Gamma_L) + S_{12} \cdot S_{21} \cdot \Gamma_L| = |1 - S_{22} \cdot \Gamma_L|$$

- determinant of the S matrix  $\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$

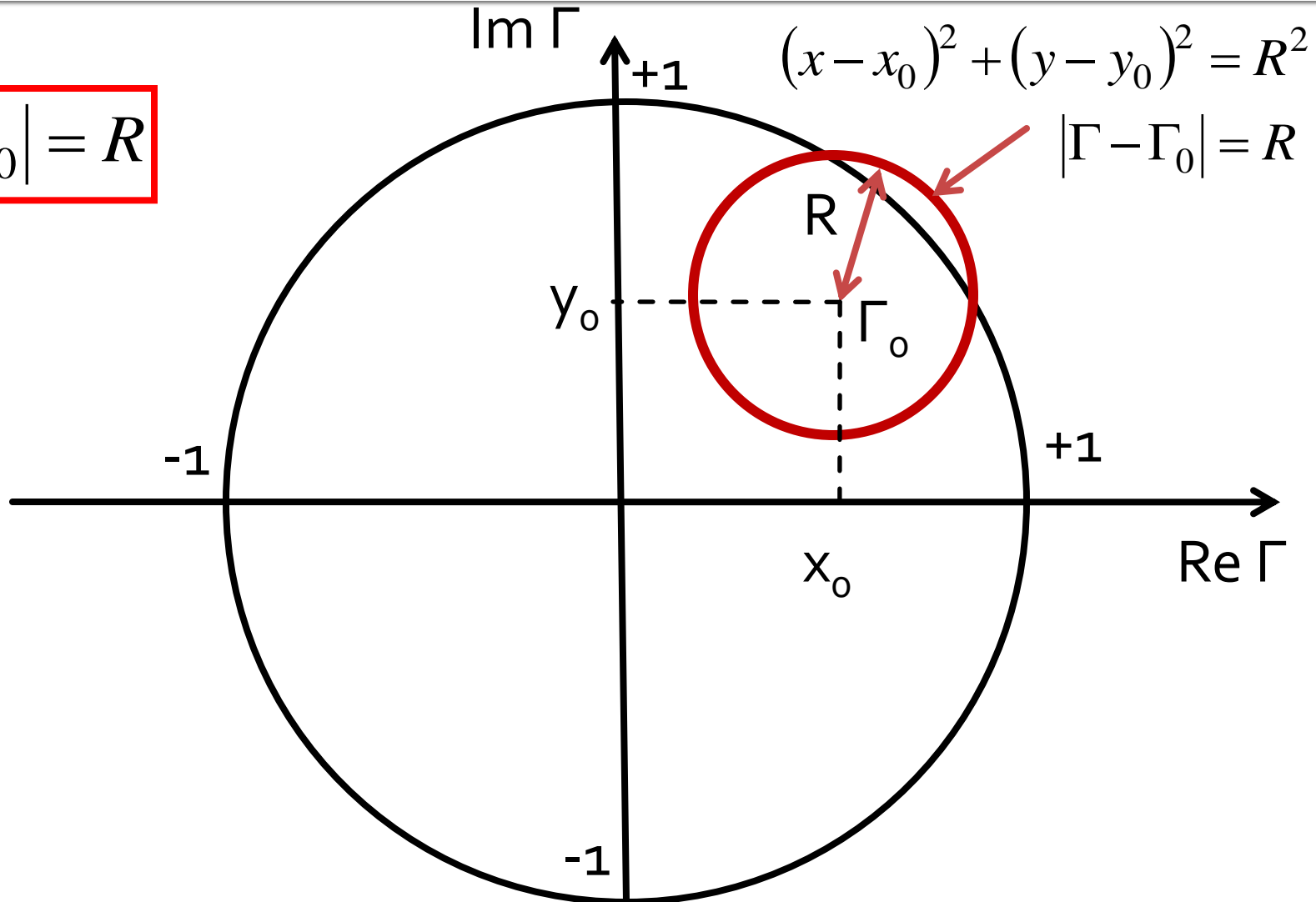
$$|S_{11} - \Delta \cdot \Gamma_L| = |1 - S_{22} \cdot \Gamma_L|$$

$$|S_{11} - \Delta \cdot \Gamma_L|^2 = |1 - S_{22} \cdot \Gamma_L|^2$$



# Stability

$$|\Gamma - \Gamma_0| = R$$



# Output stability circle (CSOUT)

$$\left| \Gamma_L - \frac{(S_{22} - \Delta \cdot S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12} \cdot S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \quad |\Gamma_L - C_L| = R_L$$

- We obtain the equation of a circle in the complex plane, which represents the locus of  $\Gamma_L$  for the **limit between stability and instability** ( $|\Gamma_{in}| = 1$ )
- This circle is the **output stability circle** ( $\Gamma_L$ )

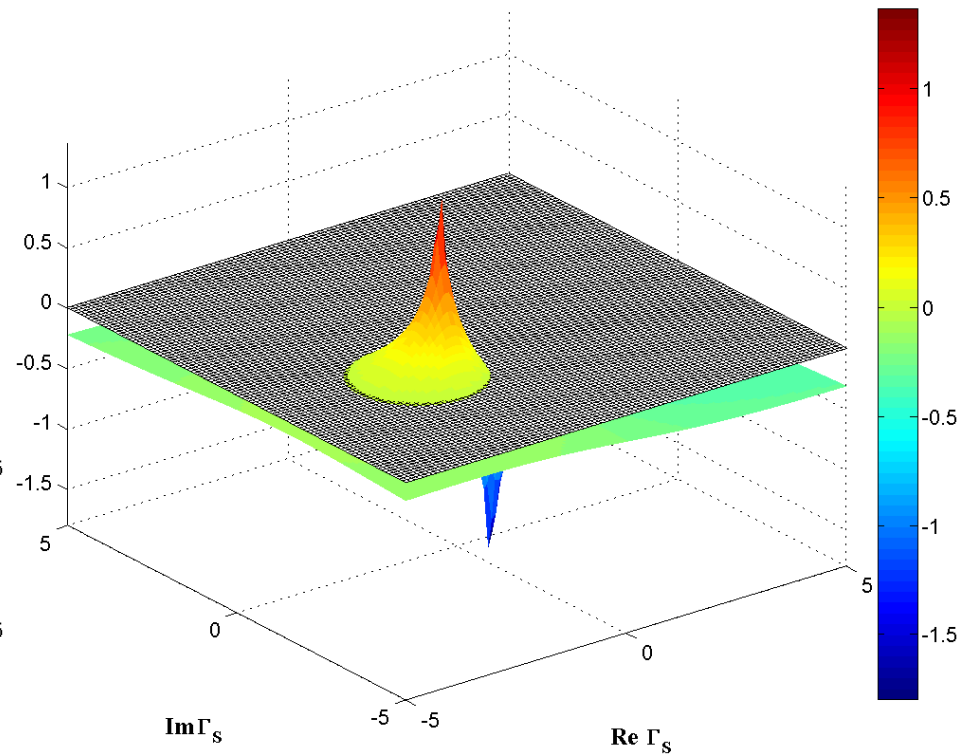
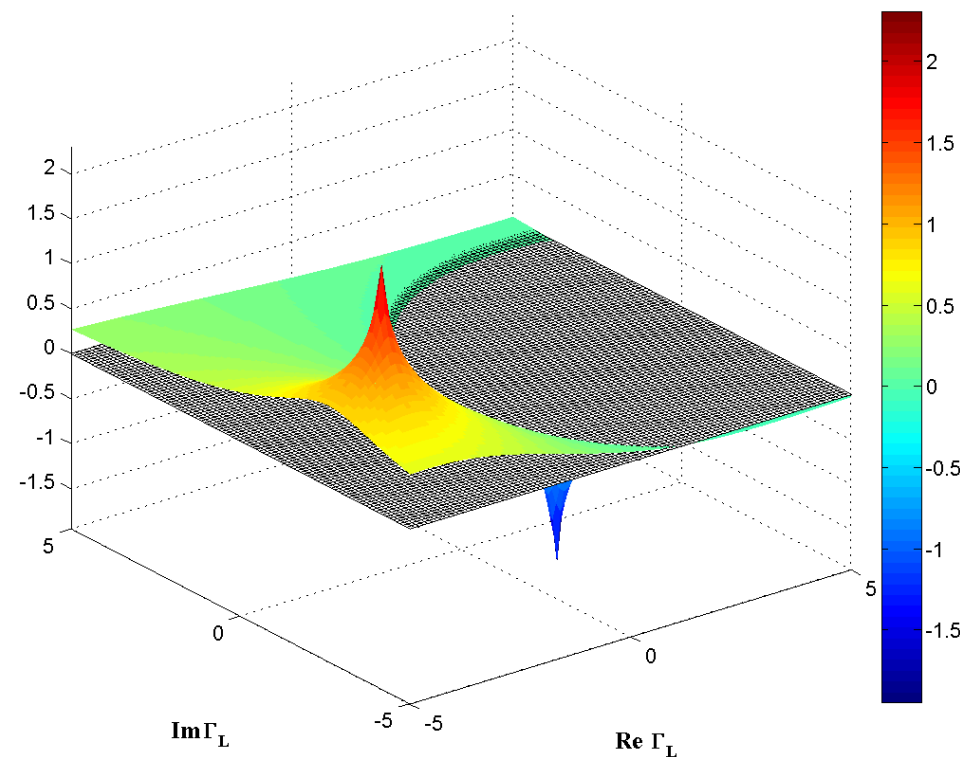
$$C_L = \frac{(S_{22} - \Delta \cdot S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \quad R_L = \frac{|S_{12} \cdot S_{21}|}{\left| |S_{22}|^2 - |\Delta|^2 \right|}$$

# 3D representation of $|\Gamma_{in}|$ , $|\Gamma_{out}|$ , $|\Gamma|=1$

- $|\Gamma| = 1 \rightarrow \log_{10}|\Gamma| = 0$ , the intersection with the plane  $z = 0$  is a circle

$\log(\Gamma_{in}(\Gamma_L))$

$\log(\Gamma_{out}(\Gamma_S))$



# Rollet's condition

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|}$$

$$\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$$

- The two-port is **unconditionally stable** if:
- two conditions are simultaneously satisfied:
  - $K > 1$
  - $|\Delta| < 1$
- together with the implicit conditions:
  - $|S_{11}| < 1$
  - $|S_{22}| < 1$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|} > 1$$

$$|\Delta| = |S_{11} \cdot S_{22} - S_{12} \cdot S_{21}| < 1$$

# $\mu$ Criterion

- Rollet's condition cannot be used to compare the relative stability of two or more devices because it involves constraints on two separate parameters,  $K$  and  $\Delta$

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta \cdot S_{11}^*| + |S_{12} \cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
  - $\mu > 1$
- together with the implicit conditions:
  - $|S_{11}| < 1$
  - $|S_{22}| < 1$
- In addition, it can be said that larger values of  $\mu$  imply greater stability
  - $\mu$  is the distance from the center of the Smith Chart to the closest output stability circle

# $\mu'$ Criterion

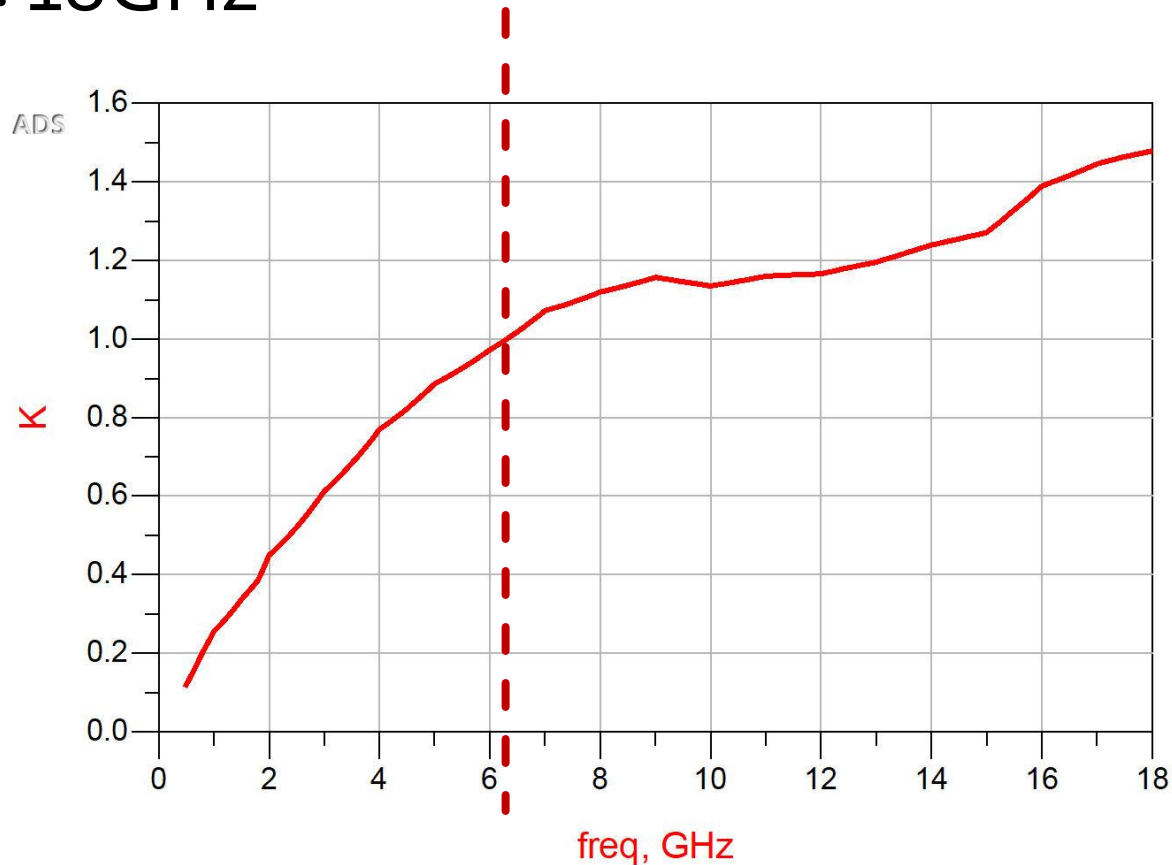
- Dual parameter to  $\mu$ , determined in relation to the input stability circles

$$\mu' = \frac{1 - |S_{22}|^2}{|S_{11} - \Delta \cdot S_{22}^*| + |S_{12} \cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
  - $\mu' > 1$
- together with the implicit conditions:
  - $|S_{11}| < 1$
  - $|S_{22}| < 1$
- In addition, it can be said that larger values of  $\mu'$  imply greater stability
  - $\mu'$  is the distance from the center of the Smith Chart to the closest input stability circle

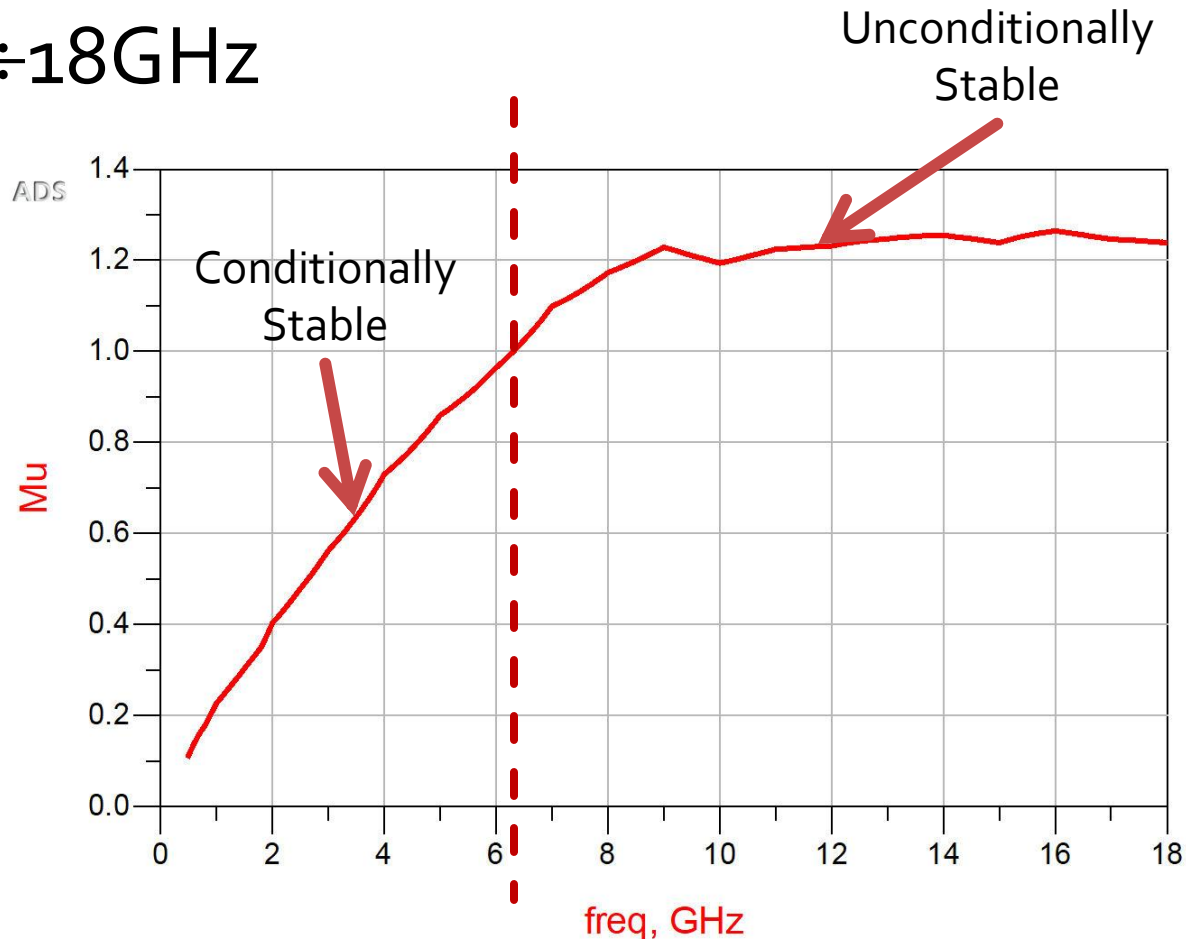
# Rollet's condition

- ATF-34143 at  $V_{ds}=3V$   $I_d=20mA$ .
- @0.5÷18GHz



# $\mu$ Criterion

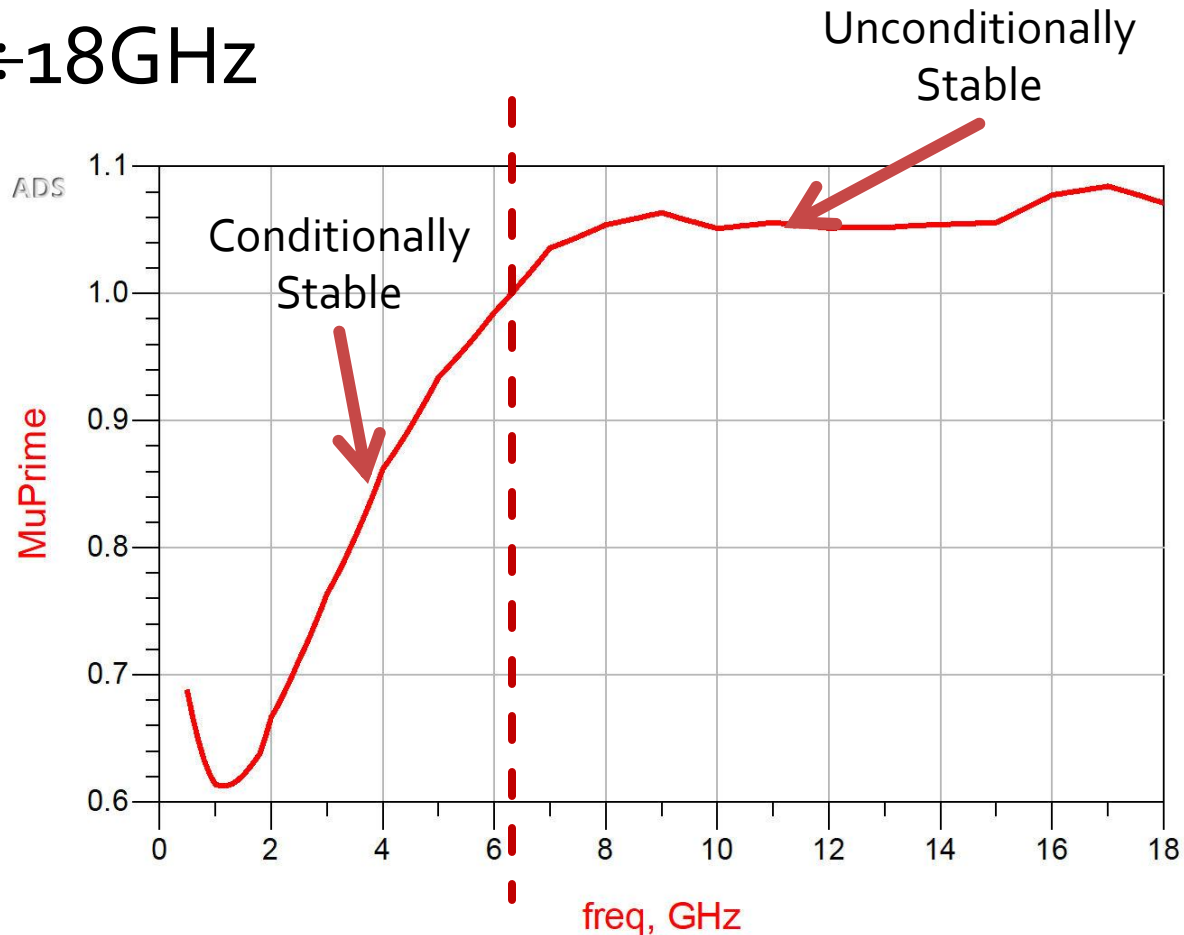
- ATF-34143 at  $V_{ds}=3V$   $I_d=20mA$ .
- @0.5÷18GHz





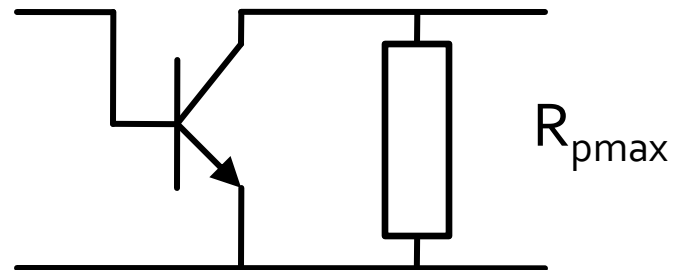
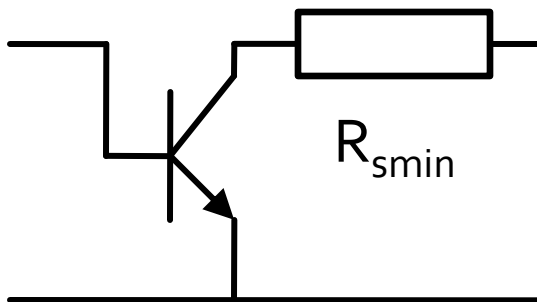
# $\mu'$ Criterion

- ATF-34143 at  $V_{ds}=3V$   $I_d=20mA$ .
- @0.5÷18GHz

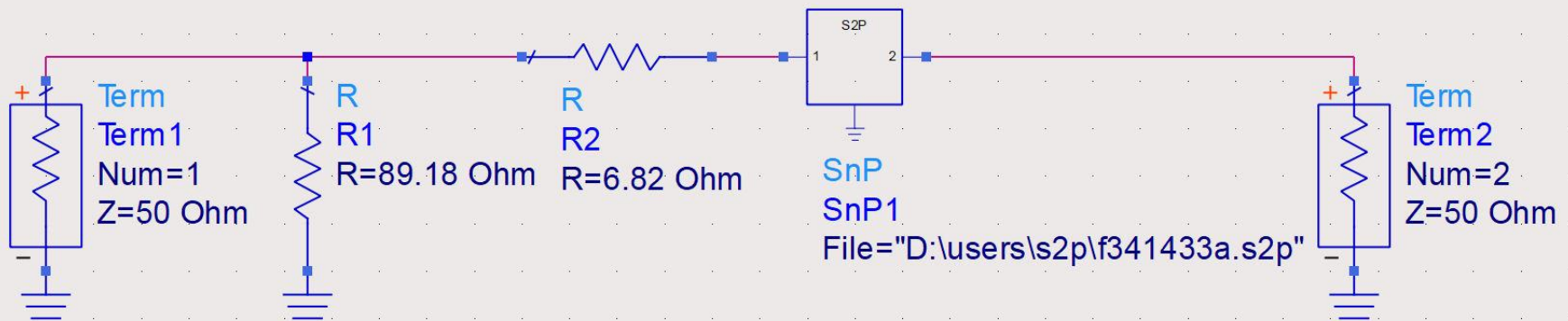


# Output series/shunt resistor

- The procedure can be applied similarly at the output (finding  $g/r$  circles tangent to CSOUT)
- From previous examples, resistive loading at the input has a positive effect over output stability and vice versa (resistive loading at the output, effect over input stability)



# Stabilization of two-port



S-PARAMETERS

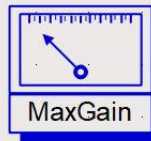
S\_Param

SP1

Start=0.5 GHz

Stop=10.0 GHz

Step=0.1 GHz

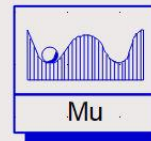


MaxGain

MaxGain

MAG

MAG=max\_gain(S)

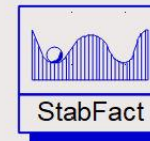


Mu

Mu

Mu1

Mu=mu(S)



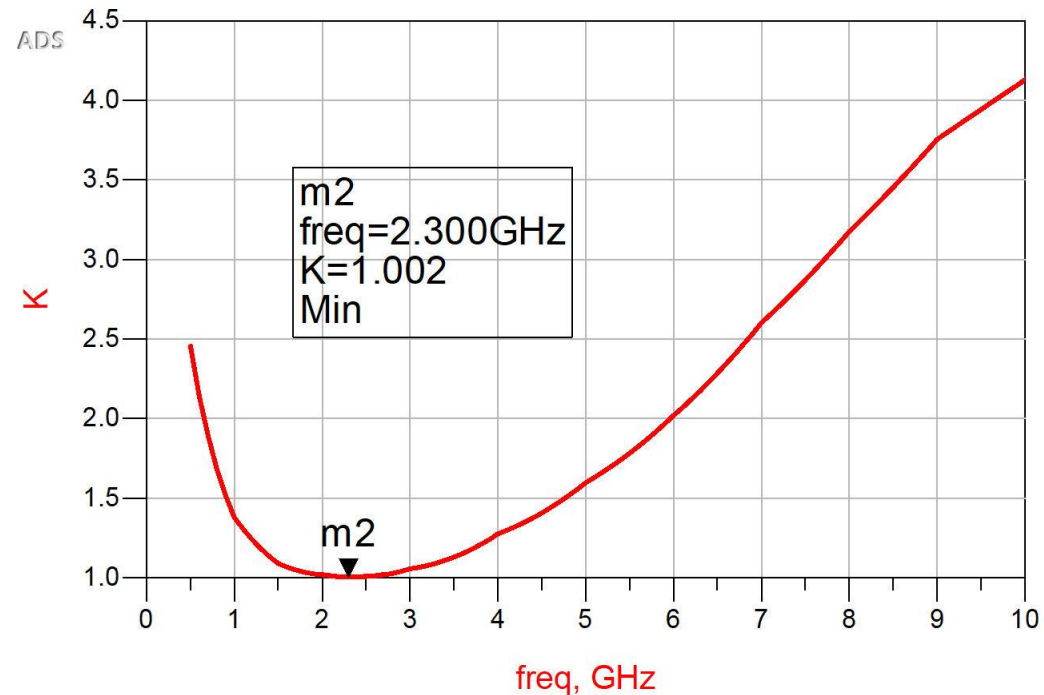
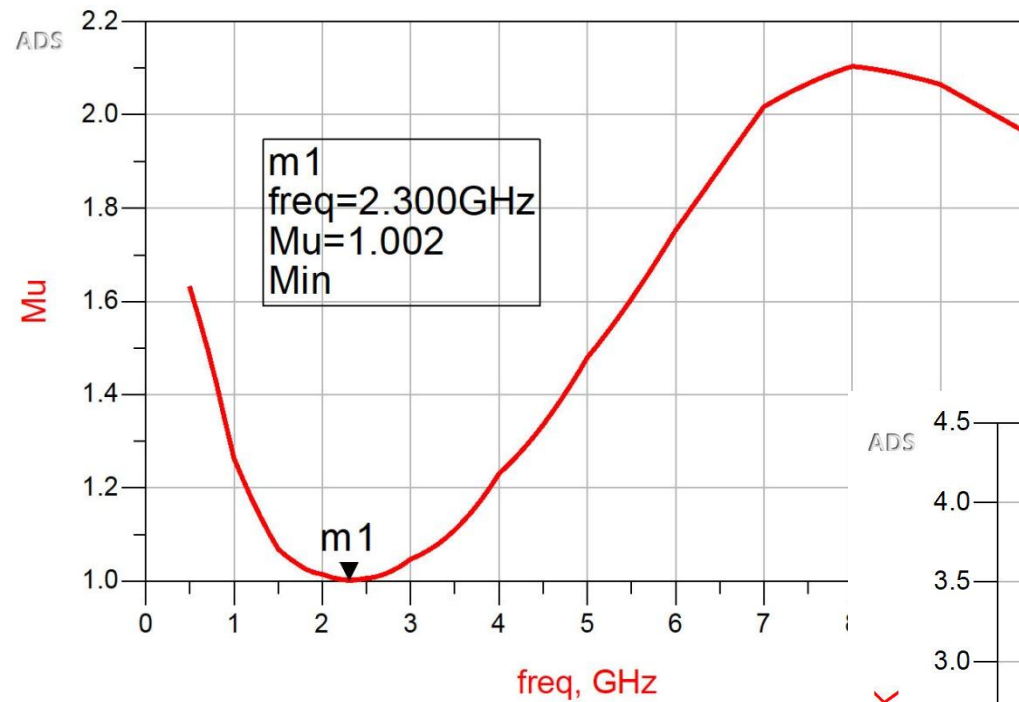
StabFact

StabFact

K

K=stab\_fact(S)

# Stabilization of two-port

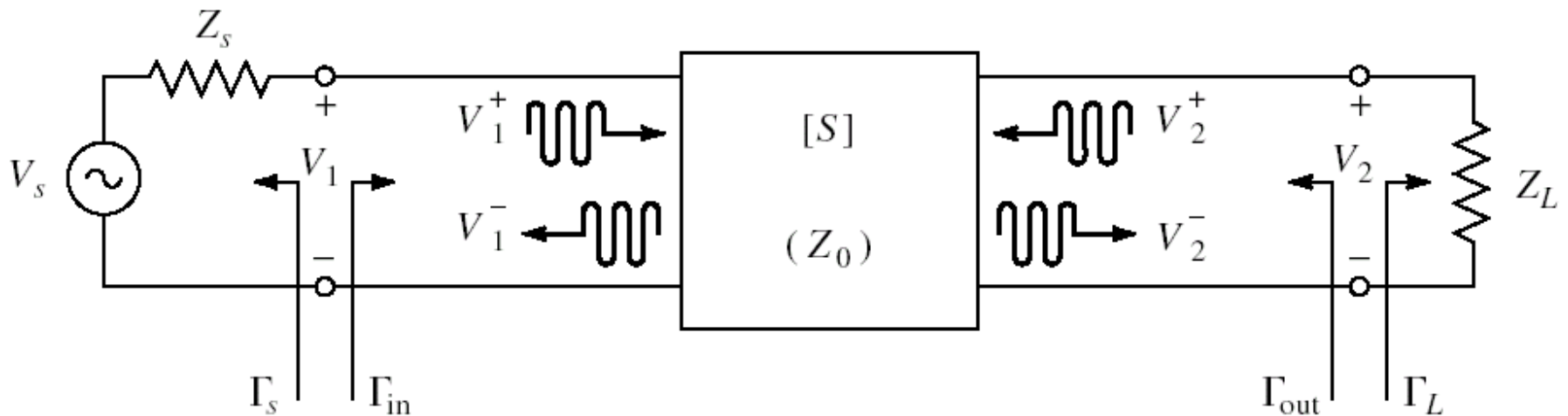


Power Gain of Microwave Amplifiers

# Microwave Amplifiers

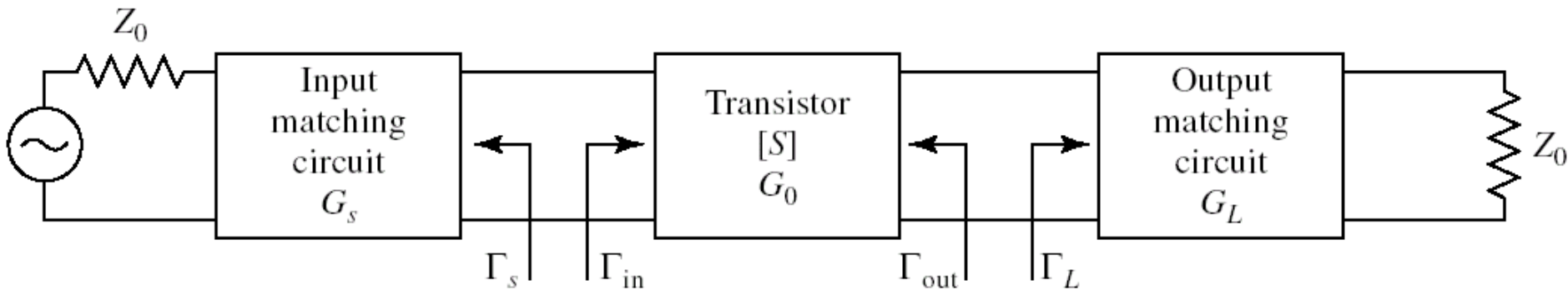
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# Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - **power gain**
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

# Design for Maximum Gain



- Maximum power gain (complex conjugate matching):

$$\Gamma_{in} = \Gamma_S^* \quad \Gamma_{out} = \Gamma_L^*$$

- For lossless matching sections

$$G_{T \max} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_S|^2) \cdot (1 - |\Gamma_L|^2)}{|1 - \Gamma_S \cdot \Gamma_{in}|^2 \cdot |1 - S_{22} \cdot \Gamma_L|^2} \quad G_{T \max} = \frac{1}{1 - |\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

- For the general case of the bilateral transistor ( $S_{12} \neq 0$ )

$\Gamma_{in}$  and  $\Gamma_{out}$  depend on each other so the input and output sections must be matched simultaneously

# Simultaneous matching

- Simultaneous matching can be achieved **if and only if** the amplifier is **unconditionally stable** at the operating frequency, and  $|\Gamma| < 1$  solutions are those with “-” sign of quadratic solutions

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1}$$

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases}$$

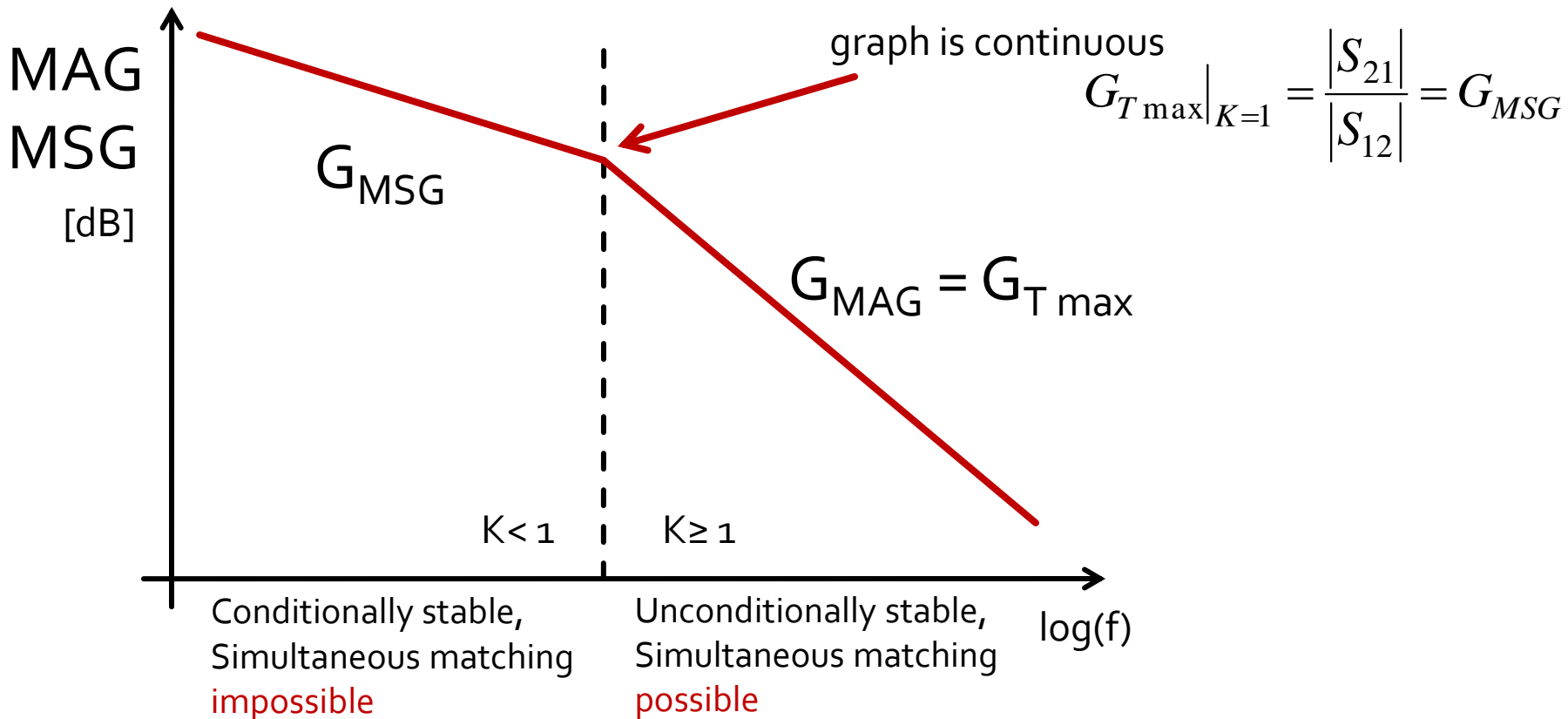
$$\Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

$$\begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$



# Maximum Available Gain

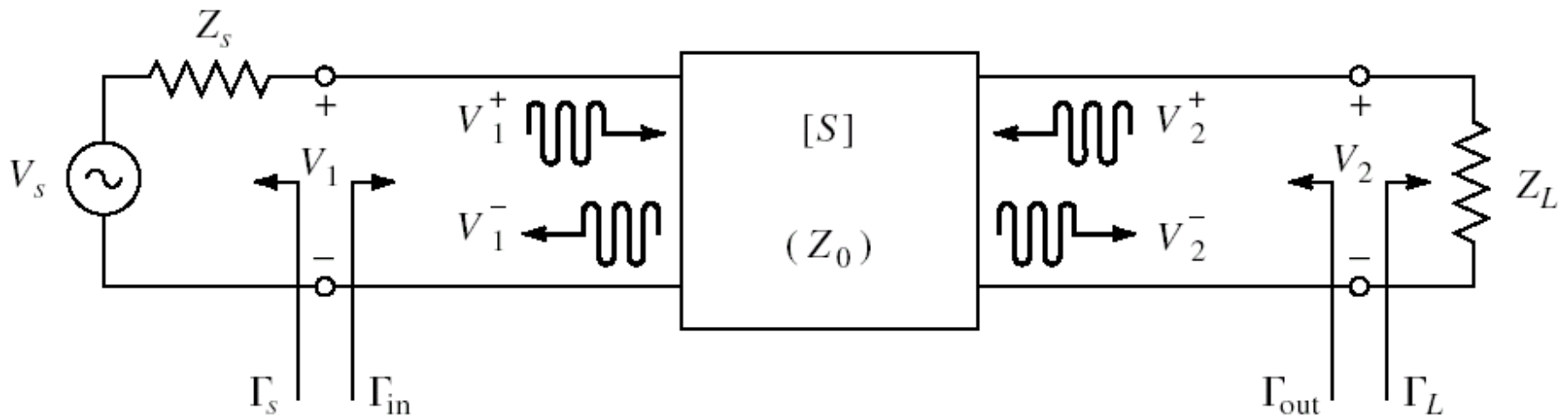
- Indicator across full frequency range of the capability to obtain a power gain



Design for Specified Gain

# Microwave Amplifiers

# Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - **power gain**
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

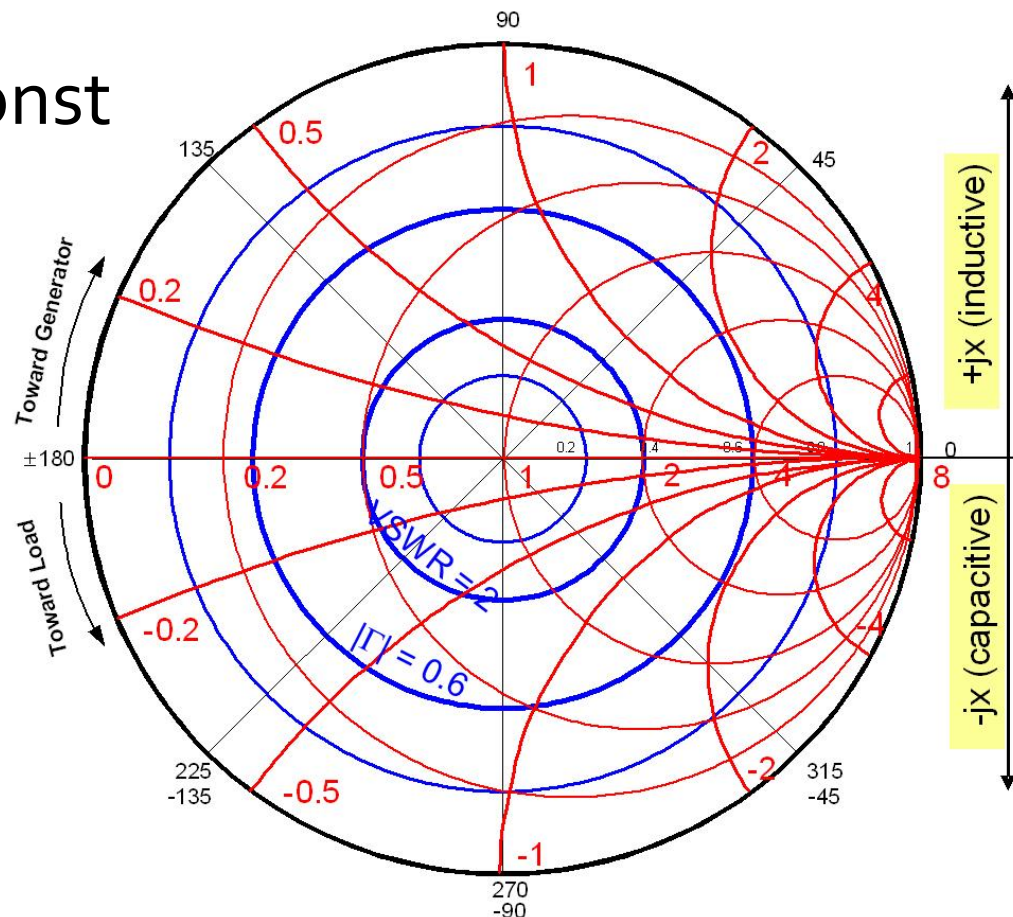
# Design for Specified Gain

- In many cases we need an approach other than “brute force” when we prefer to design for **less than the maximum obtainable gain**, in order to:
  - improve noise behavior ( $L_3 + C_9$ )
  - improve stability
  - improve VSWR
  - control performance at multiple frequencies
  - improve amplifier's bandwidth

# Constant VSWR circles

- Certain applications may require a certain ratio between maximum / minimum line voltage
- $VSWR = \text{const} \rightarrow \Gamma = \text{const}$

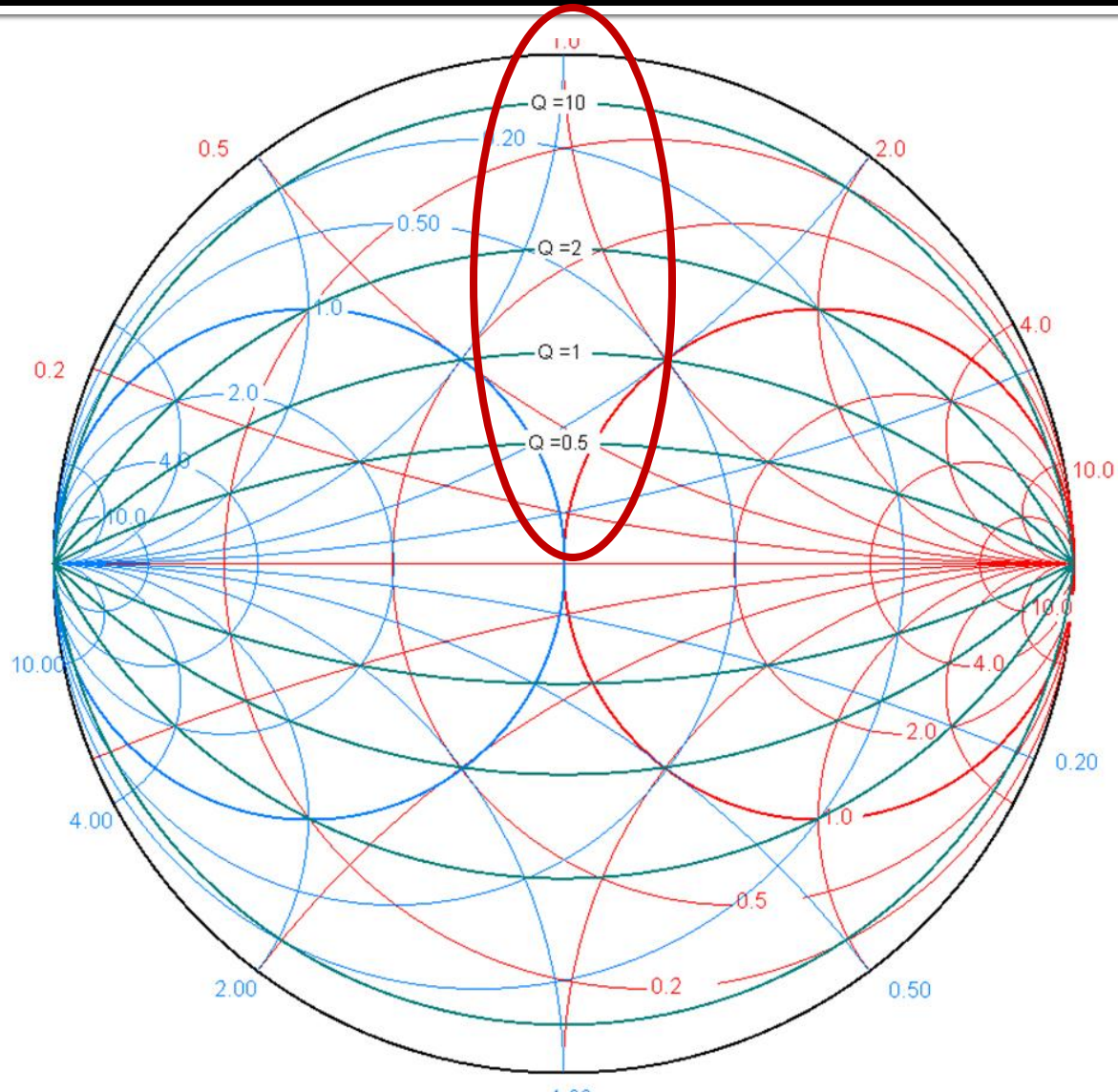
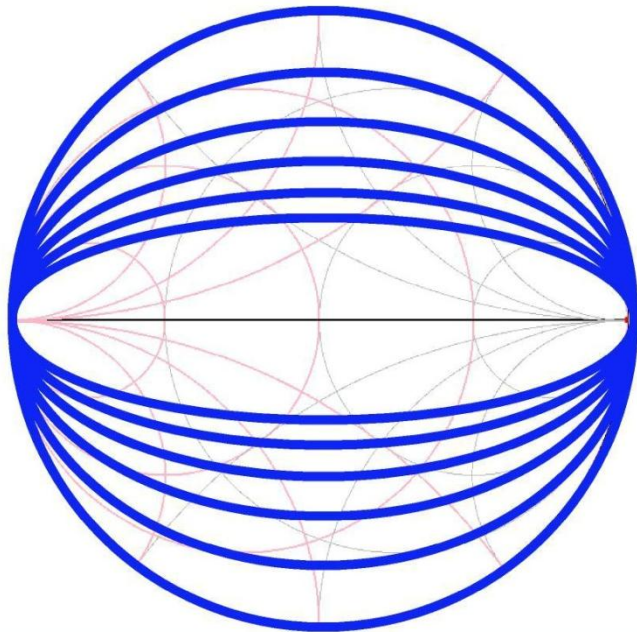
$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$



# Constant Q circles

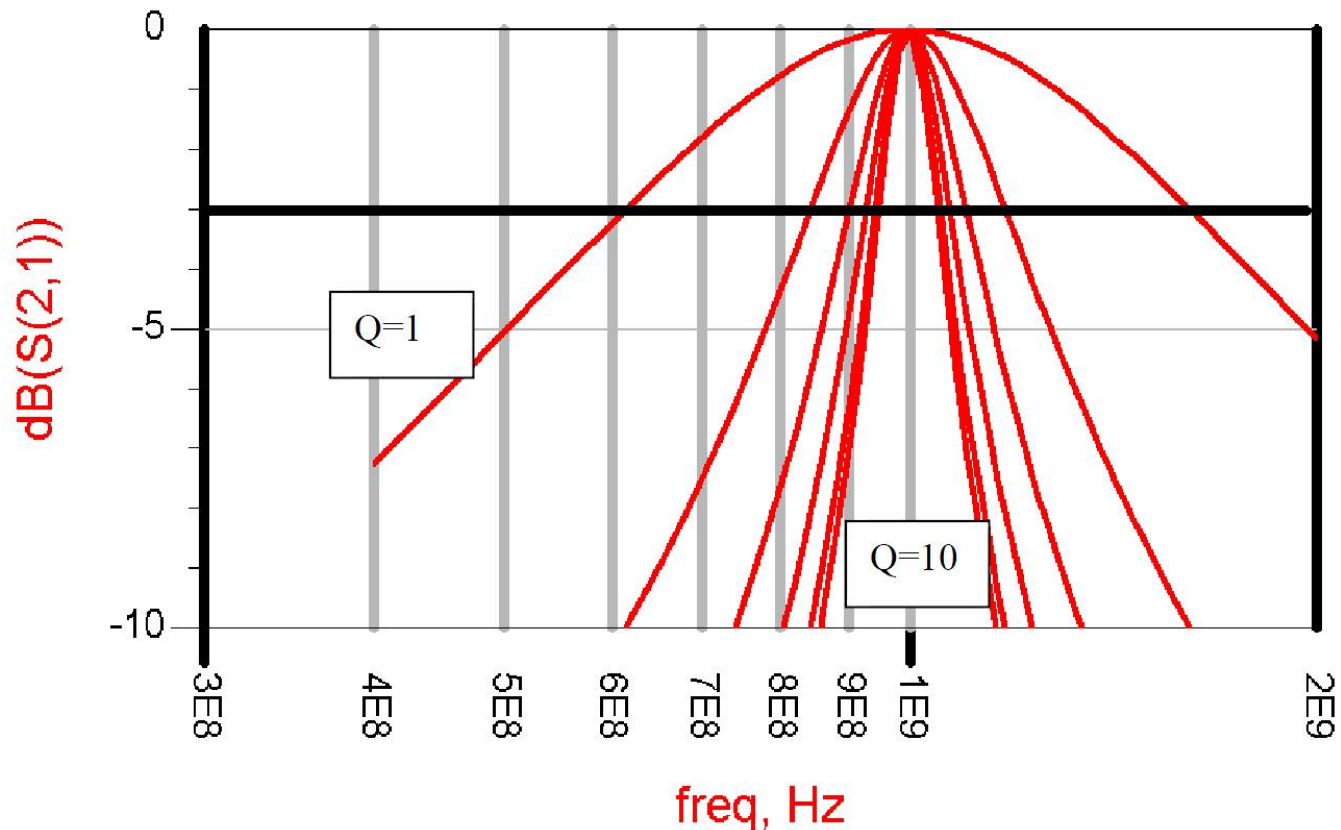
- Quality factor Q

$$Q = \frac{X}{R} = \frac{G}{B} = \text{const}$$



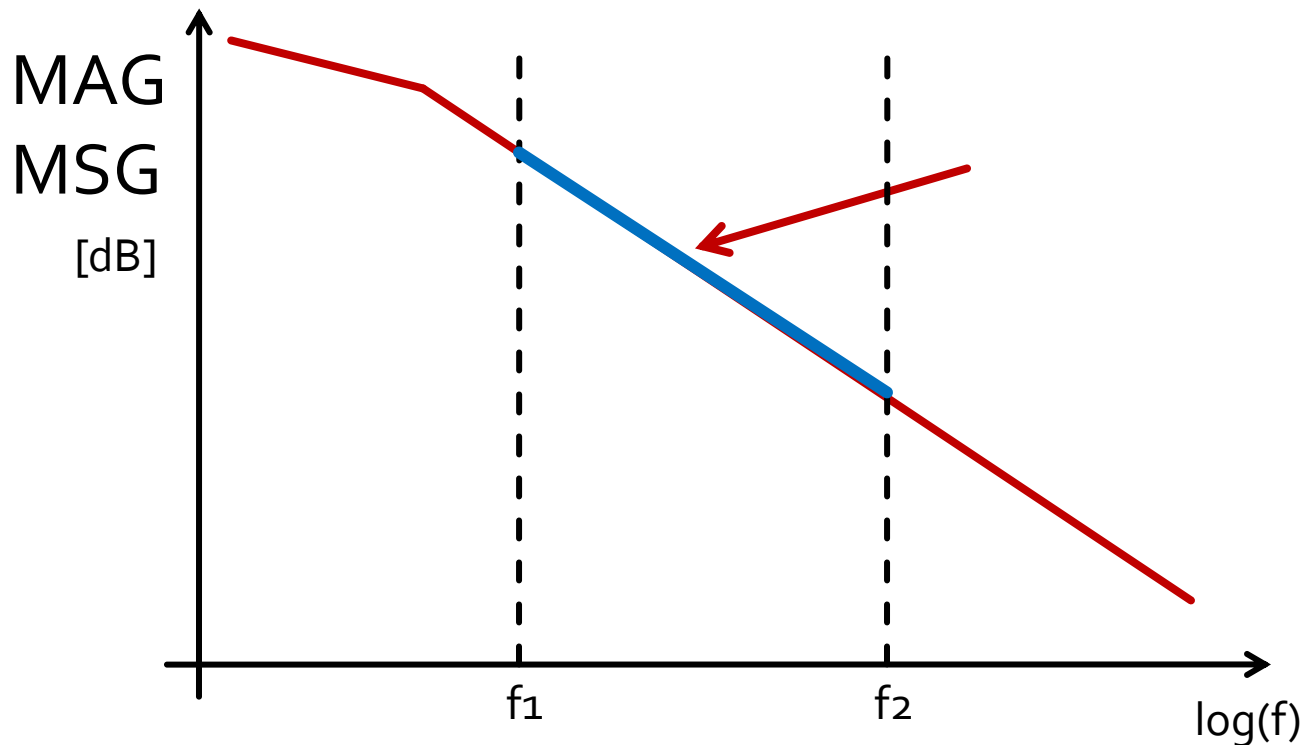
# Quality factor - bandwidth

- High quality factor is equivalent with narrow bandwidth



# Wide bandwidth amplifier

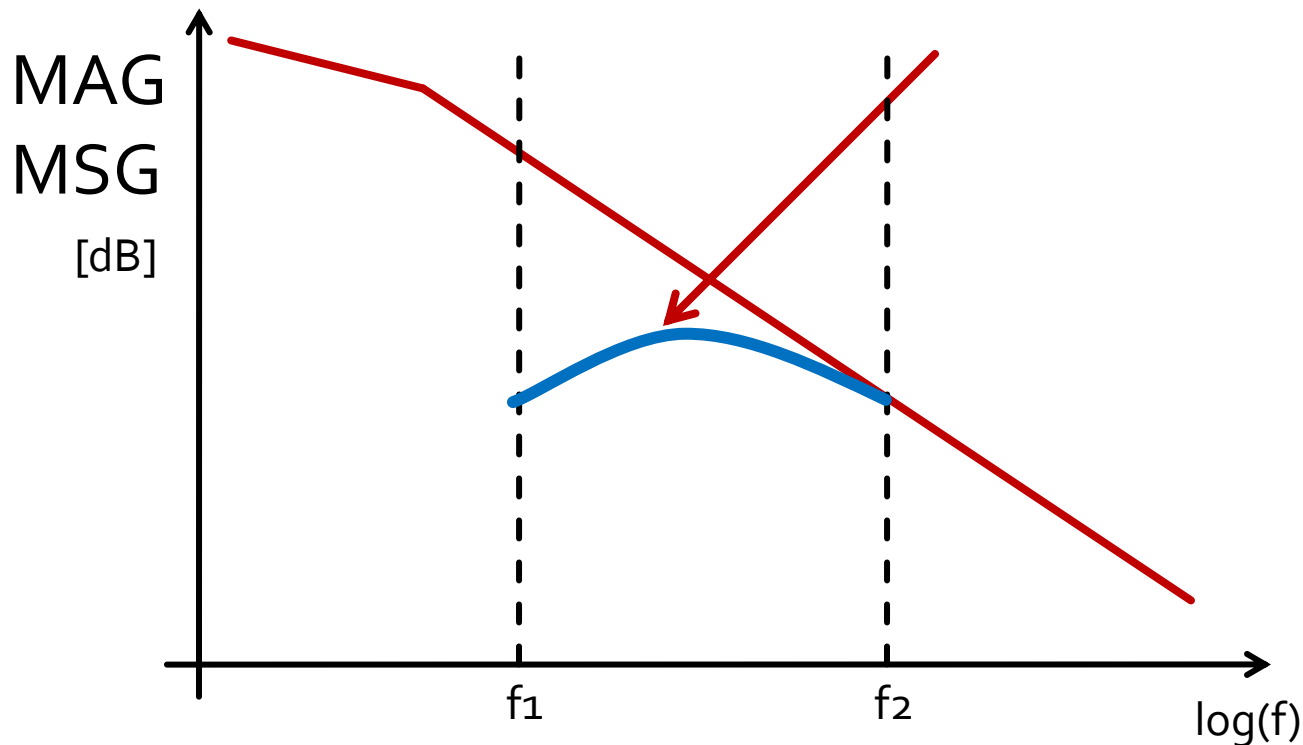
- Design for maximum gain at two different frequencies creates an frequency unbalanced amplifier





# Wide bandwidth amplifier

- Design for maximum gain at highest frequency
- Controlled mismatch at lower frequency
  - eventually at more frequencies inside the bandwidth



# Design for Specified Gain

- Assumes the amplifier device **unilateral**

Input and output can be treated independently

$$G_{TU} = |S_{21}|^2 \cdot \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2} \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

$S_{12} \cong 0$        $\Gamma_{in} = S_{11}$

- Maximum power gain

$$\Gamma_S = S_{11}^*$$

$$\Gamma_L = S_{22}^*$$

$$G_{TU \max} = \frac{1}{1 - |S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |S_{22}|^2}$$

# Unilateral figure of merit

- Allows estimation of the error introduced by the unilateral assumption

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} \quad U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{(1-|S_{11}|^2) \cdot (1-|S_{22}|^2)}$$

- We compute U then the maximum and minimum deviation of  $G_{TU}$  from  $G_T$ 
  - this deviation must be accounted in the design as a reserve gain against the target gain

$$-20 \cdot \log(1+U) < G_T [dB] - G_{TU} [dB] < -20 \cdot \log(1-U)$$

# Example

- ATF-34143 at  $V_{ds}=3V$   $I_d=20mA$ .
- @5GHz

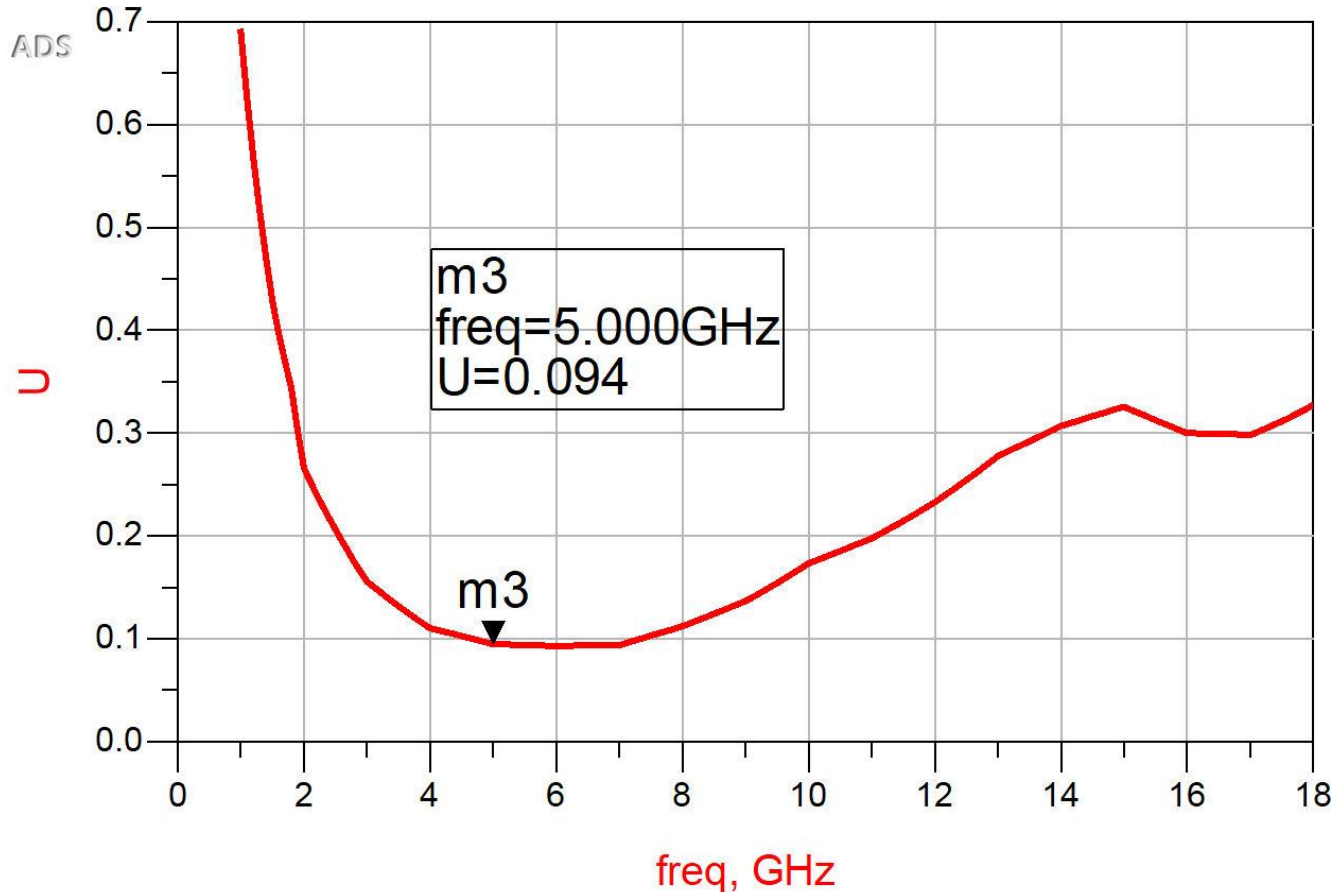
- $S_{11} = 0.64 \angle 139^\circ$
- $S_{12} = 0.119 \angle -21^\circ$
- $S_{21} = 3.165 \angle 16^\circ$
- $S_{22} = 0.22 \angle 146^\circ$

$$U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{(1 - |S_{11}|^2) \cdot (1 - |S_{22}|^2)} = 0.094$$

$$-0.783 \text{ dB} < G_T [\text{dB}] - G_{TU} [\text{dB}] < 0.861 \text{ dB}$$

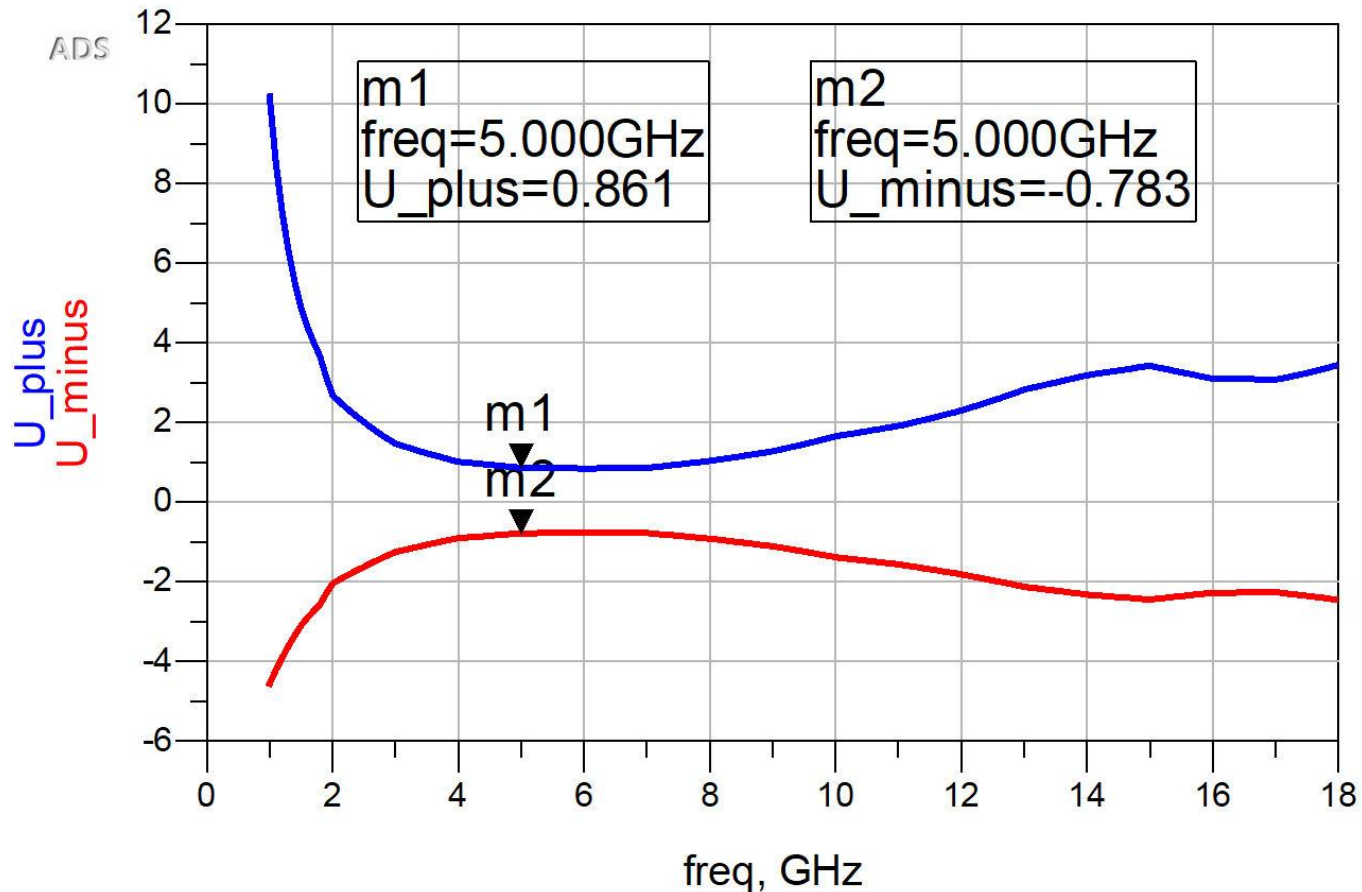
# Example

- ATF-34143 at  $V_{ds}=3V$   $I_d=20mA$ .
- @5GHz

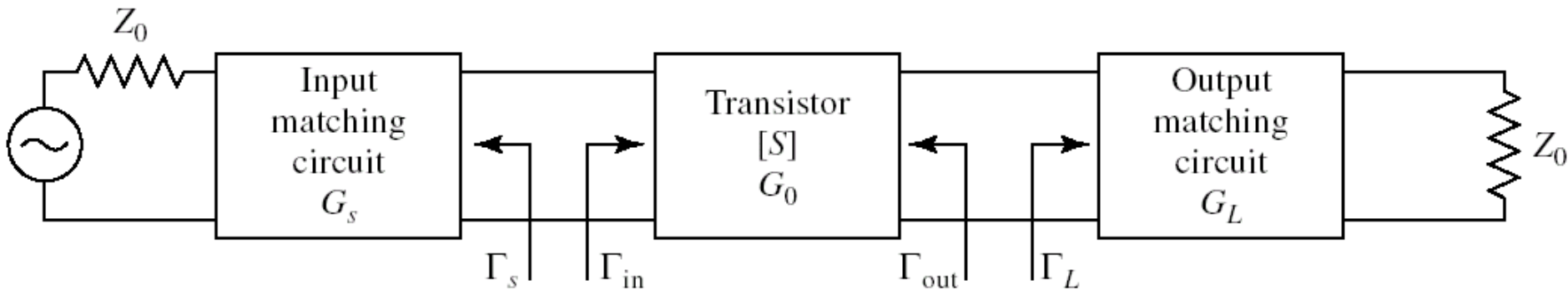


# Example

- ATF-34143 at  $V_{ds}=3V$   $I_d=20mA$ .
- @5GHz



# Design for Specified Gain



- In the unilateral assumption:

$$G_{TU} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2}$$

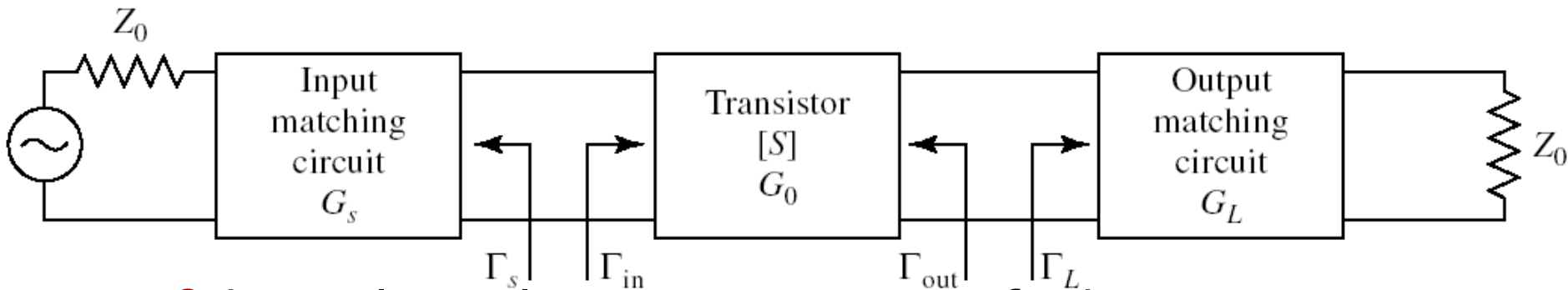
$$G_S = G_S(\Gamma_S)$$

$$G_0 = |S_{21}|^2$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

$$G_L = G_L(\Gamma_L)$$

# Design for Specified Gain



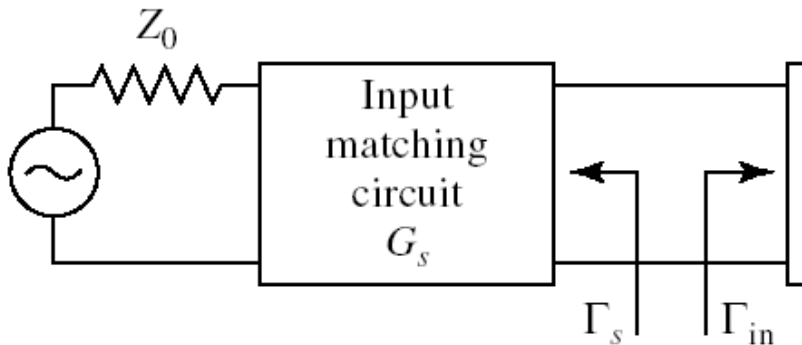
- **If** the unilateral assumption is justified :
  - power gain added by the input matching circuit **is not** influenced by the output matching circuit  $G_S = G_S(\Gamma_S)$
  - power gain added by the output matching circuit **is not** influenced by the input matching circuit  $G_L = G_L(\Gamma_L)$
- Output /Input match can be designed independently
  - We can impose different demands for input/output
  - Total gain is:

$$G_T = G_S \cdot G_0 \cdot G_L$$

$$G_T [dB] = G_S [dB] + G_0 [dB] + G_L [dB]$$



# Input matching circuit



$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2}$$

- Maximum gain in the case of complex conjugate match

$$\Gamma_S = S_{11}^* \Rightarrow G_{S \max} = \frac{1}{1 - |S_{11}|^2}$$

- For any other input matching circuit:

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2} < G_{S \max} = \frac{1}{1 - |S_{11}|^2}$$

# Example

- ATF-34143 at  $V_{ds}=3V$   $I_d=20mA$ .
- @5GHz

- $S_{11} = 0.64 \angle 139^\circ$
- $S_{12} = 0.119 \angle -21^\circ$
- $S_{21} = 3.165 \angle 16^\circ$
- $S_{22} = 0.22 \angle 146^\circ$

$$U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{(1 - |S_{11}|^2) \cdot (1 - |S_{22}|^2)} = 0.094$$

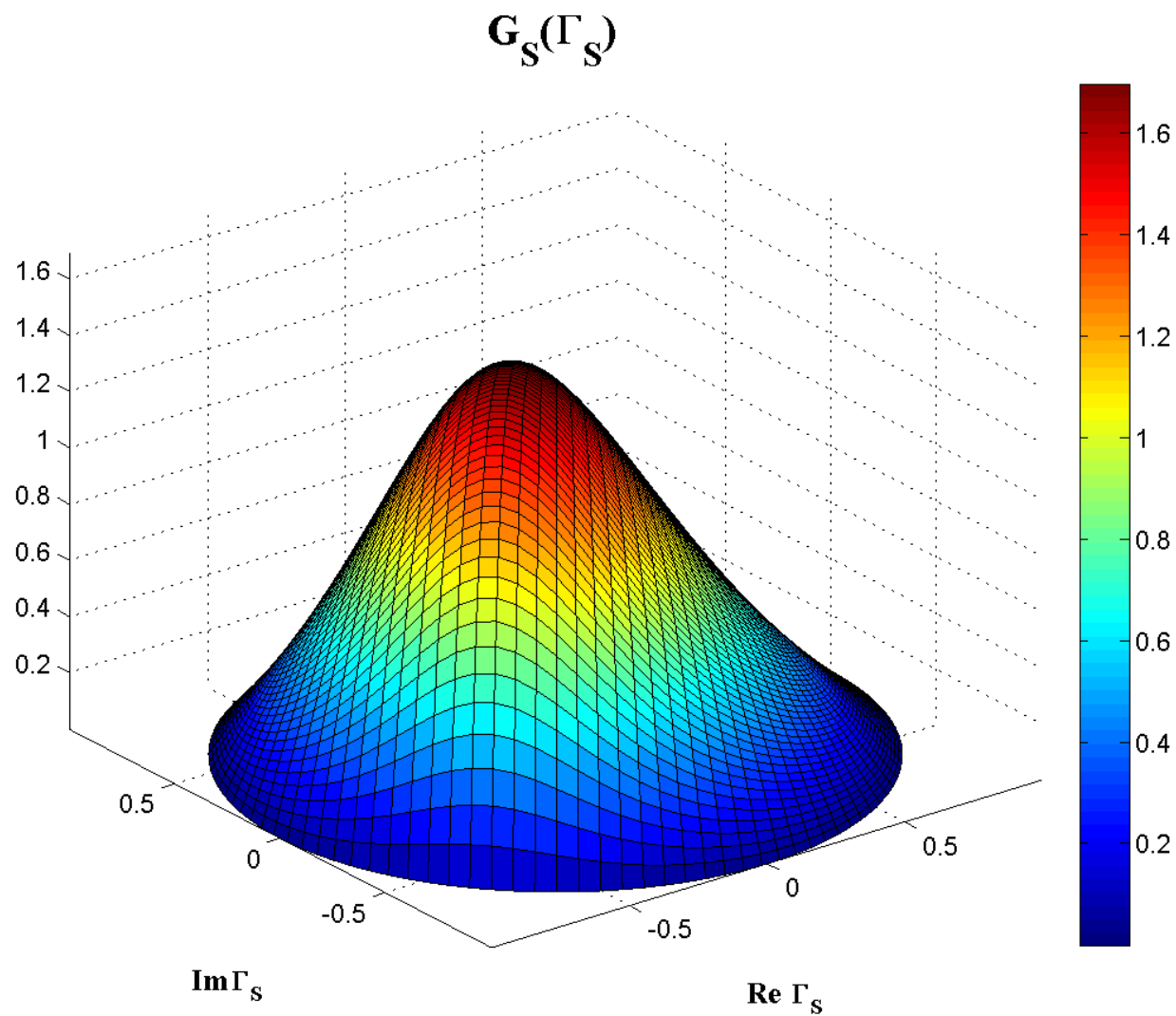
$$-0.783 \text{ dB} < G_T [\text{dB}] - G_{TU} [\text{dB}] < 0.861 \text{ dB}$$

$$G_{TU \max} = \frac{1}{1 - |S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |S_{22}|^2} = 17.83$$

$$G_{TU \max} [\text{dB}] = 12.511 \text{ dB}$$

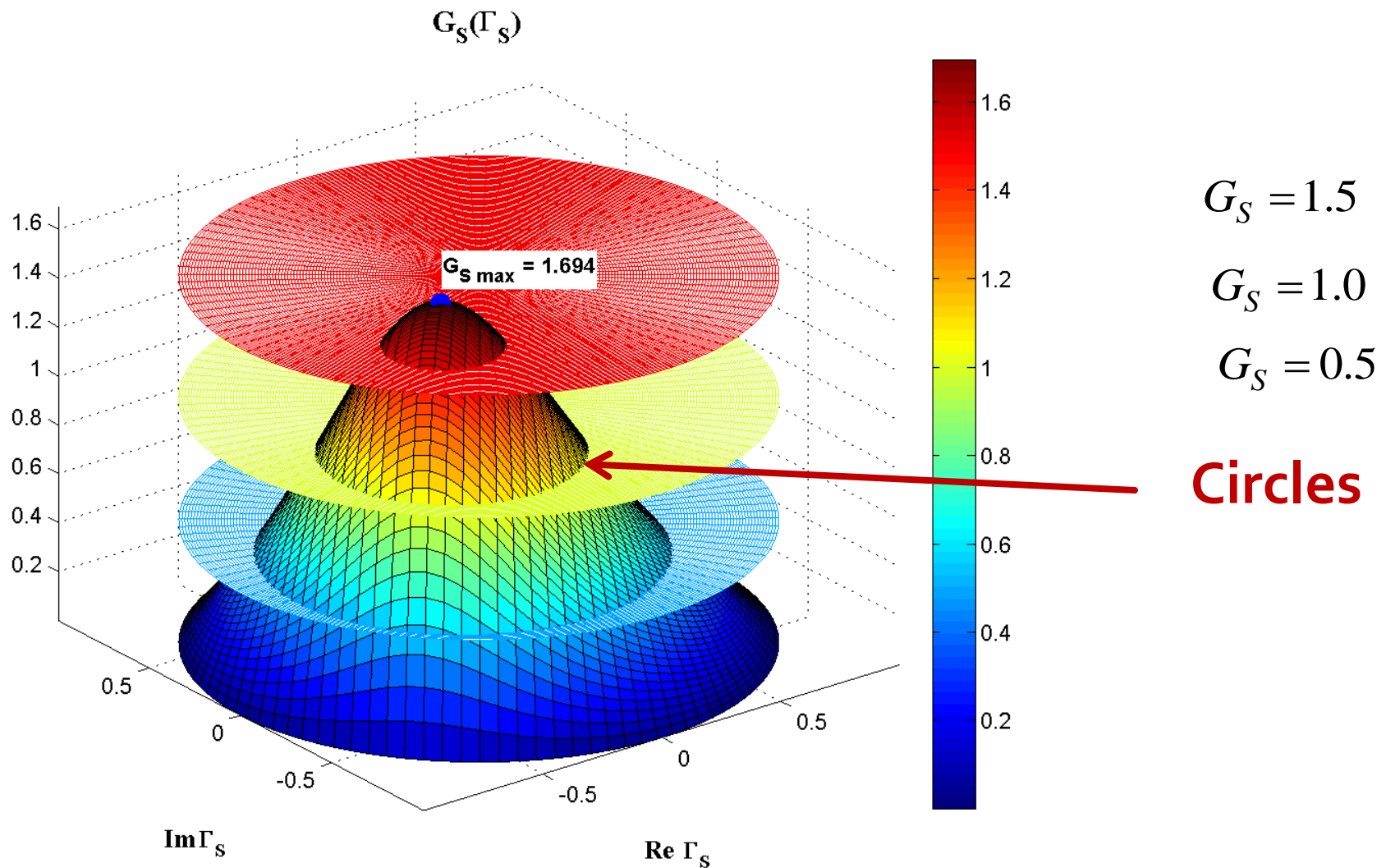
$$G_{S \max} = \frac{1}{1 - |S_{11}|^2} = 1.694 = 2.289 \text{ dB}$$

# $G_S(\Gamma_S)$

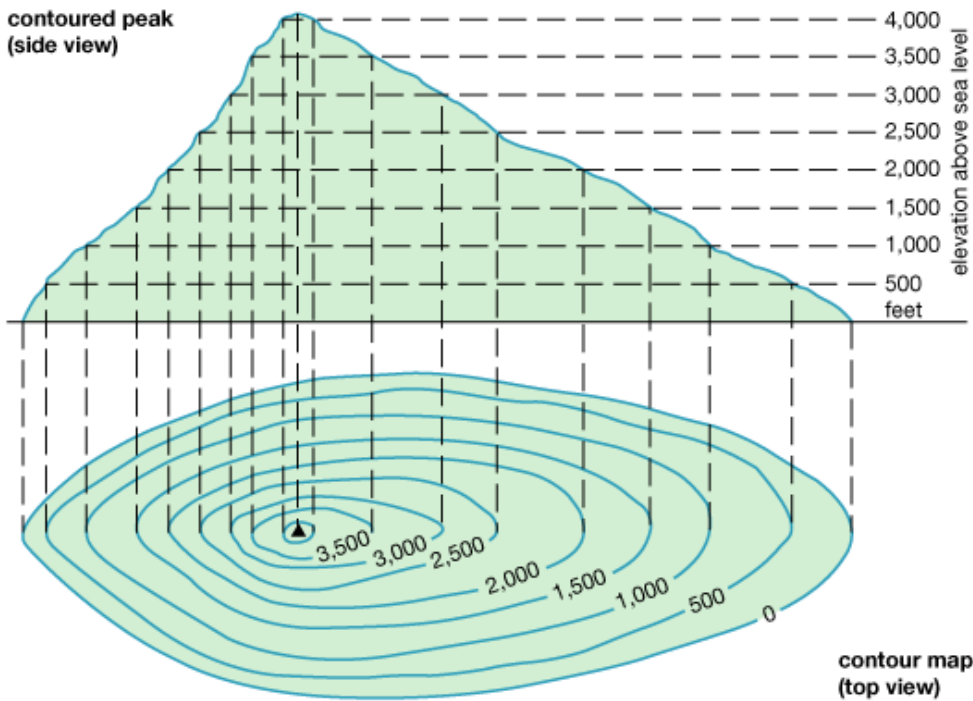


$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2}$$

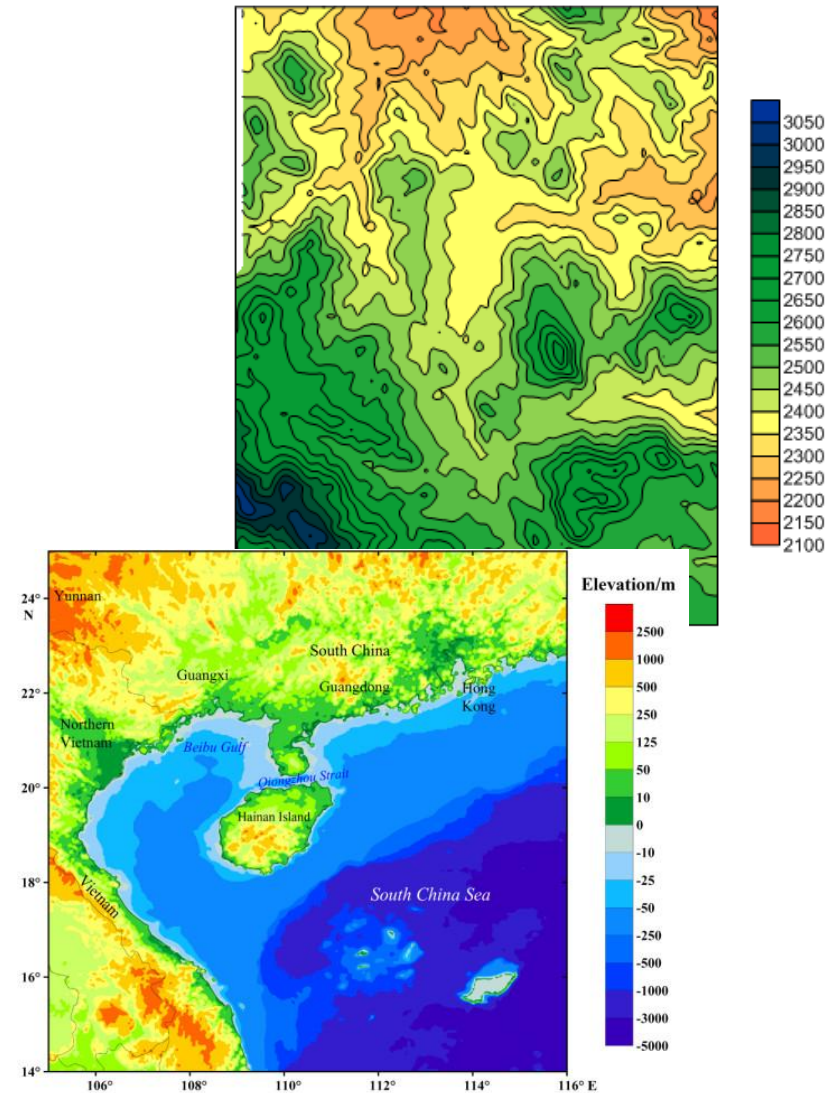
# $G_S(\Gamma_S)$ , constant value contours



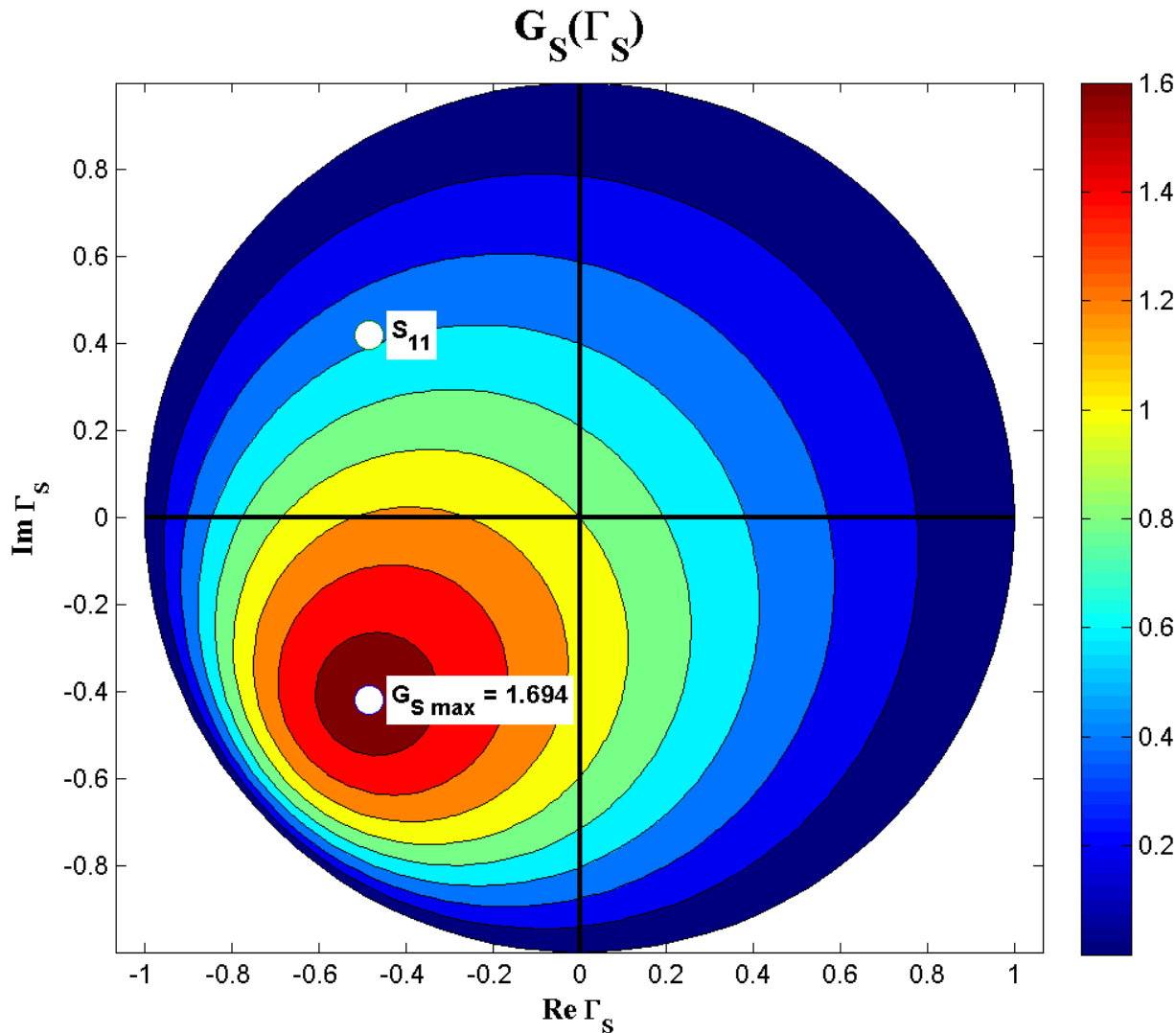
# Contour map/lines



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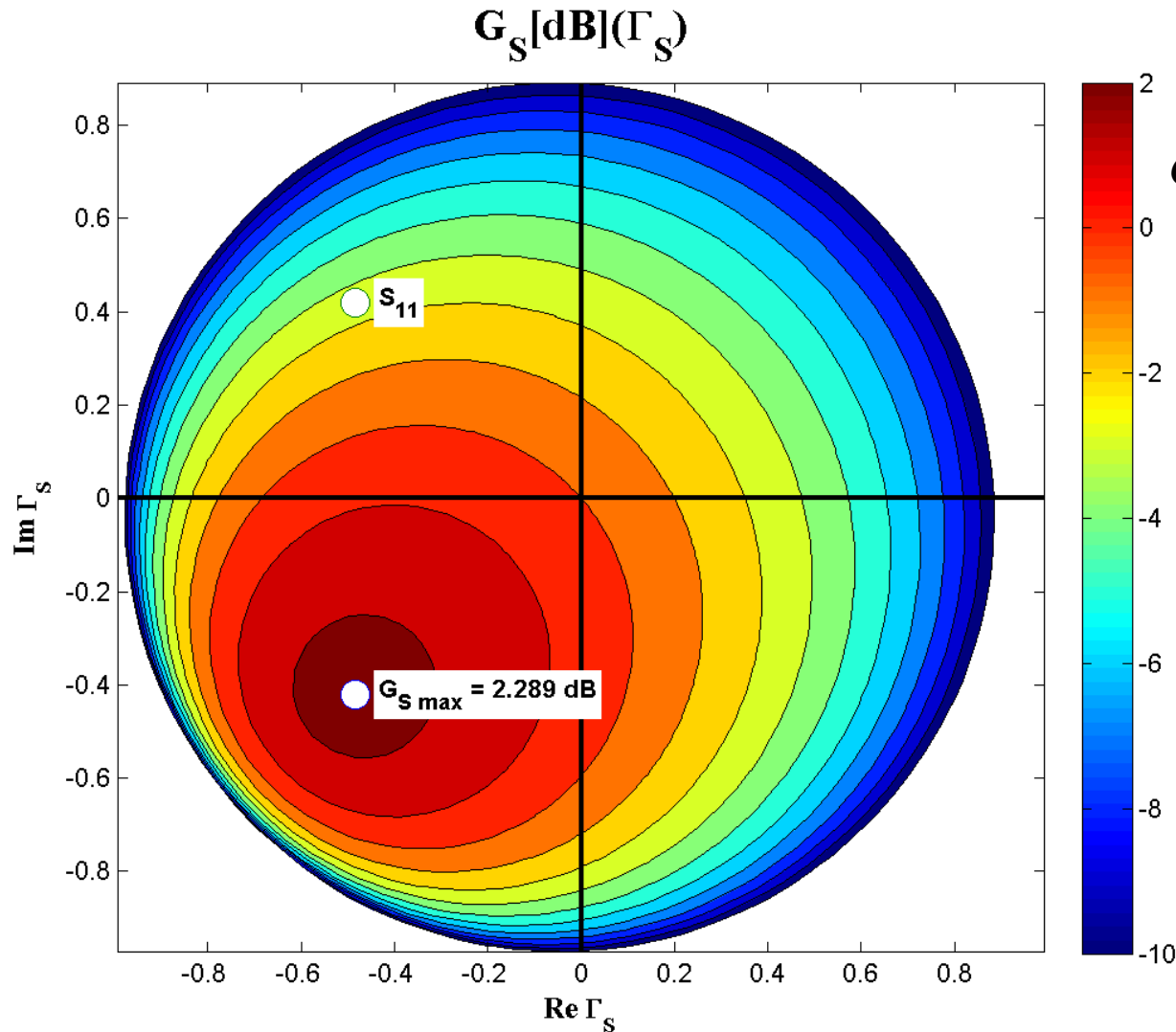
# $G_S(\Gamma_S)$ , constant value contours



$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2}$$

$$G_{S \max} = G_S \Big|_{\Gamma_S = S_{11}^*}$$

# $G_S[dB](\Gamma_S)$ , constant value contours



$$G_S[dB] = 10 \cdot \log \left( \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2} \right)$$

$$G_{S \max} = G_S \Big|_{\Gamma_S = S_{11}^*}$$

# Input section constant gain circles

- The normalized gain factor (**linear scale!**)

$$g_S = \frac{G_S}{G_{S_{\max}}} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2} \cdot (1 - |S_{11}|^2) < 1$$

- Locus of the points with fixed values  $g_S < 1$

$$g_S \cdot |1 - S_{11} \cdot \Gamma_S|^2 = (1 - |\Gamma_S|^2) \cdot (1 - |S_{11}|^2)$$

$$(g_S \cdot |S_{11}|^2 + 1 - |S_{11}|^2) \cdot |\Gamma_S|^2 - g_S \cdot (S_{11} \cdot \Gamma_S + S_{11}^* \cdot \Gamma_S^*) = 1 - |S_{11}|^2 - g_S$$

$$\Gamma_S \cdot \Gamma_S^* - \frac{g_S \cdot (S_{11} \cdot \Gamma_S + S_{11}^* \cdot \Gamma_S^*)}{1 - (1 - g_S) \cdot |S_{11}|^2} = \frac{1 - |S_{11}|^2 - g_S}{1 - (1 - g_S) \cdot |S_{11}|^2} + \frac{g_S^2 \cdot |S_{11}|^2}{[1 - (1 - g_S) \cdot |S_{11}|^2]^2}$$



# Input section constant gain circles

$$\left| \Gamma_S - \frac{g_S \cdot S_{11}^*}{1 - (1 - g_S) \cdot |S_{11}|^2} \right| = \frac{\sqrt{1 - g_S} \cdot (1 - |S_{11}|^2)}{1 - (1 - g_S) \cdot |S_{11}|^2} \quad |\Gamma_S - C_S| = R_S$$

$$C_S = \frac{g_S \cdot S_{11}^*}{1 - (1 - g_S) \cdot |S_{11}|^2} \quad R_S = \frac{\sqrt{1 - g_S} \cdot (1 - |S_{11}|^2)}{1 - (1 - g_S) \cdot |S_{11}|^2}$$

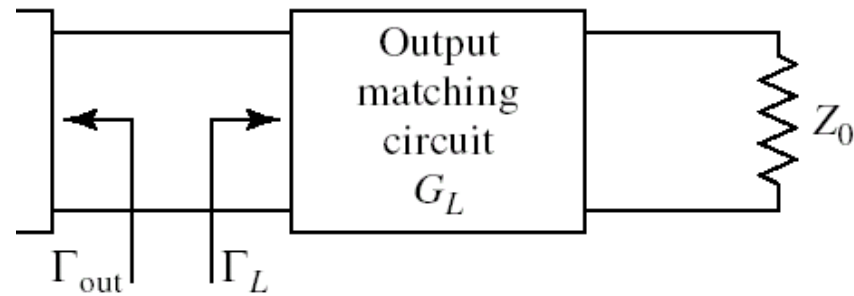
- Equation of a circle in the complex plane where  $\Gamma_S$  is plotted
- **Interpretation:** Any reflection coefficient  $\Gamma_S$  which plotted in the complex plane lies **on** the circle drawn for  $g_{\text{circle}} = G_{\text{circle}}/G_{S_{\text{max}}}$  will lead to a gain  $G_S = G_{\text{circle}}$ 
  - Any reflection coefficient  $\Gamma_S$  plotted **outside** this circle will lead to a gain  $G_S < G_{\text{circle}}$
  - Any reflection coefficient  $\Gamma_S$  plotted **inside** this circle will lead to a gain  $G_S > G_{\text{circle}}$

# Input section constant gain circles

$$C_S = \frac{g_S \cdot S_{11}^*}{1 - (1 - g_S) \cdot |S_{11}|^2} \quad R_S = \frac{\sqrt{1 - g_S} \cdot (1 - |S_{11}|^2)}{1 - (1 - g_S) \cdot |S_{11}|^2}$$

- The centers of each family of circles lie along straight lines given by the angle of  $\Gamma_{S_{\max}} = S_{11}^*$
- Circles are plotted (traditionally, CAD) in **logarithmic scale** ([dB])
  - formulas are in **linear scale!**
- The circle for  $G_S = 0$  dB will always pass through the origin of the complex plane (center of the Smith chart)

# Output section constant gain circles



$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

- Maximum gain for  $\Gamma_L = S_{22}^* \Rightarrow G_{Lmax} = \frac{1}{1 - |S_{22}|^2}$

$$g_L = \frac{G_L}{G_{Lmax}} = \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2} \cdot (1 - |S_{22}|^2) < 1$$

- Similar computations

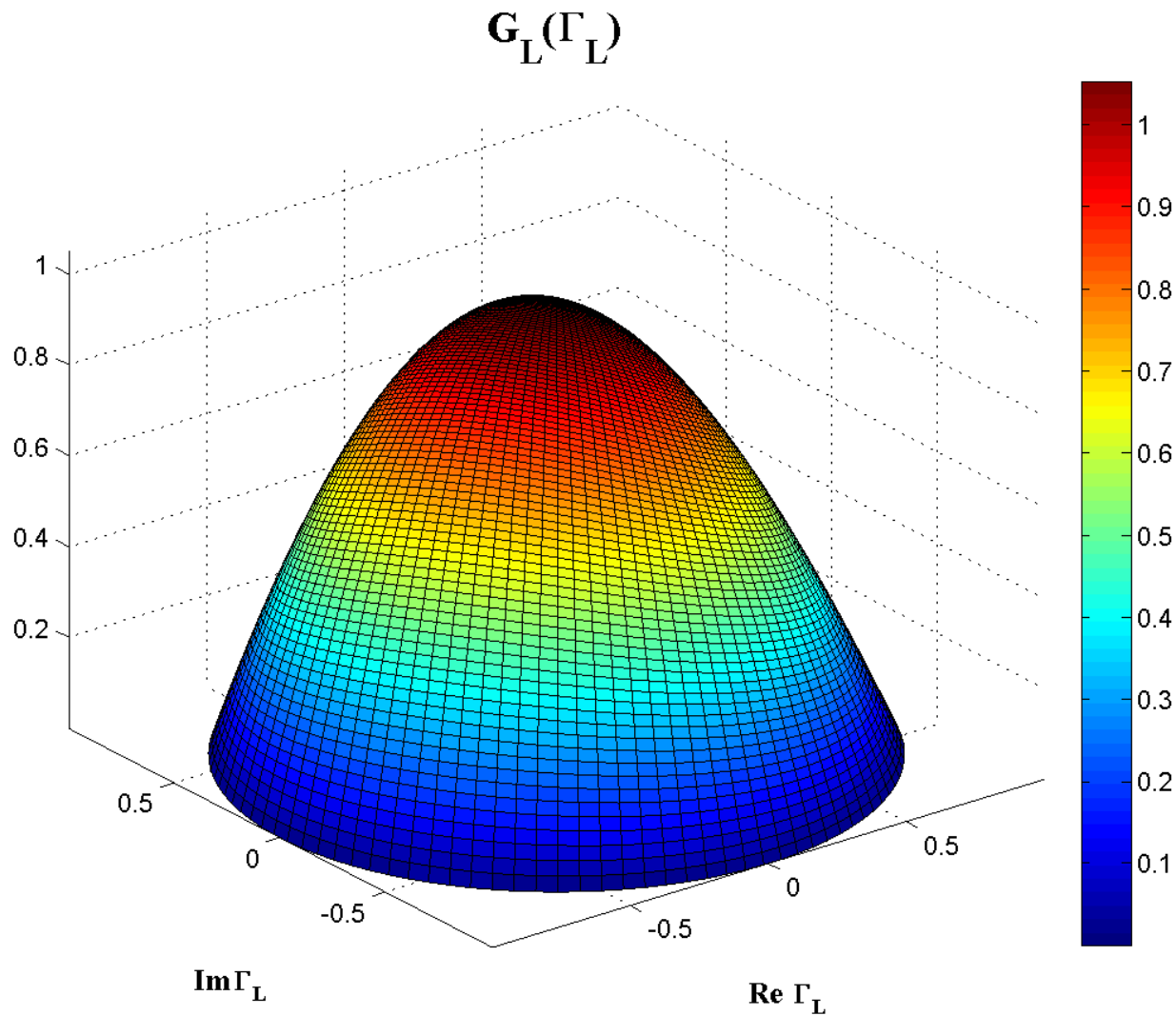
$$C_L = \frac{g_L \cdot S_{22}^*}{1 - (1 - g_L) \cdot |S_{22}|^2}$$

$$R_L = \frac{\sqrt{1 - g_L} \cdot (1 - |S_{22}|^2)}{1 - (1 - g_L) \cdot |S_{22}|^2}$$

- Example

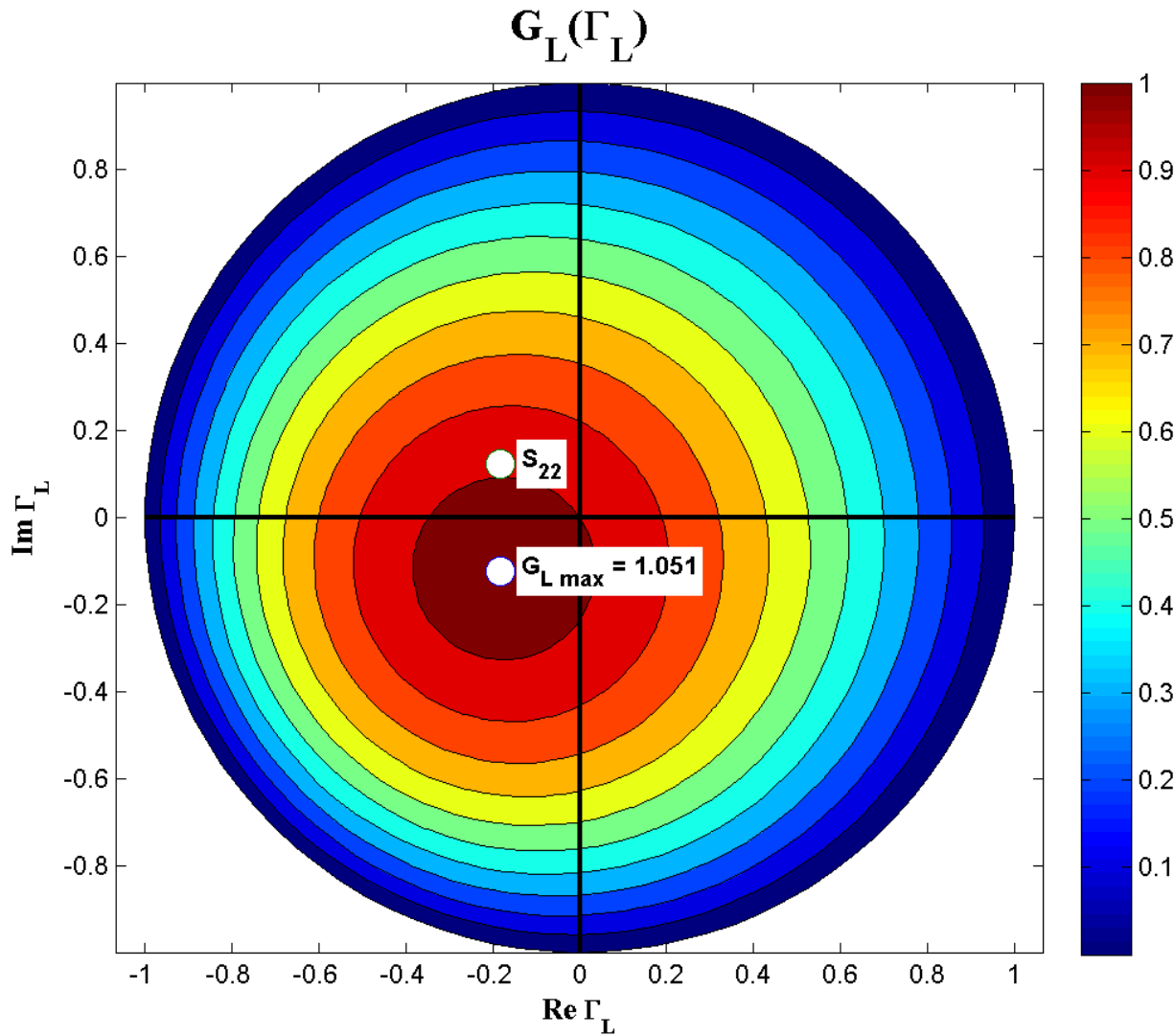
$$G_{Lmax} = \frac{1}{1 - |S_{22}|^2} = 1.051 = 0.215 \text{ dB}$$

# $G_L(\Gamma_L)$



$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

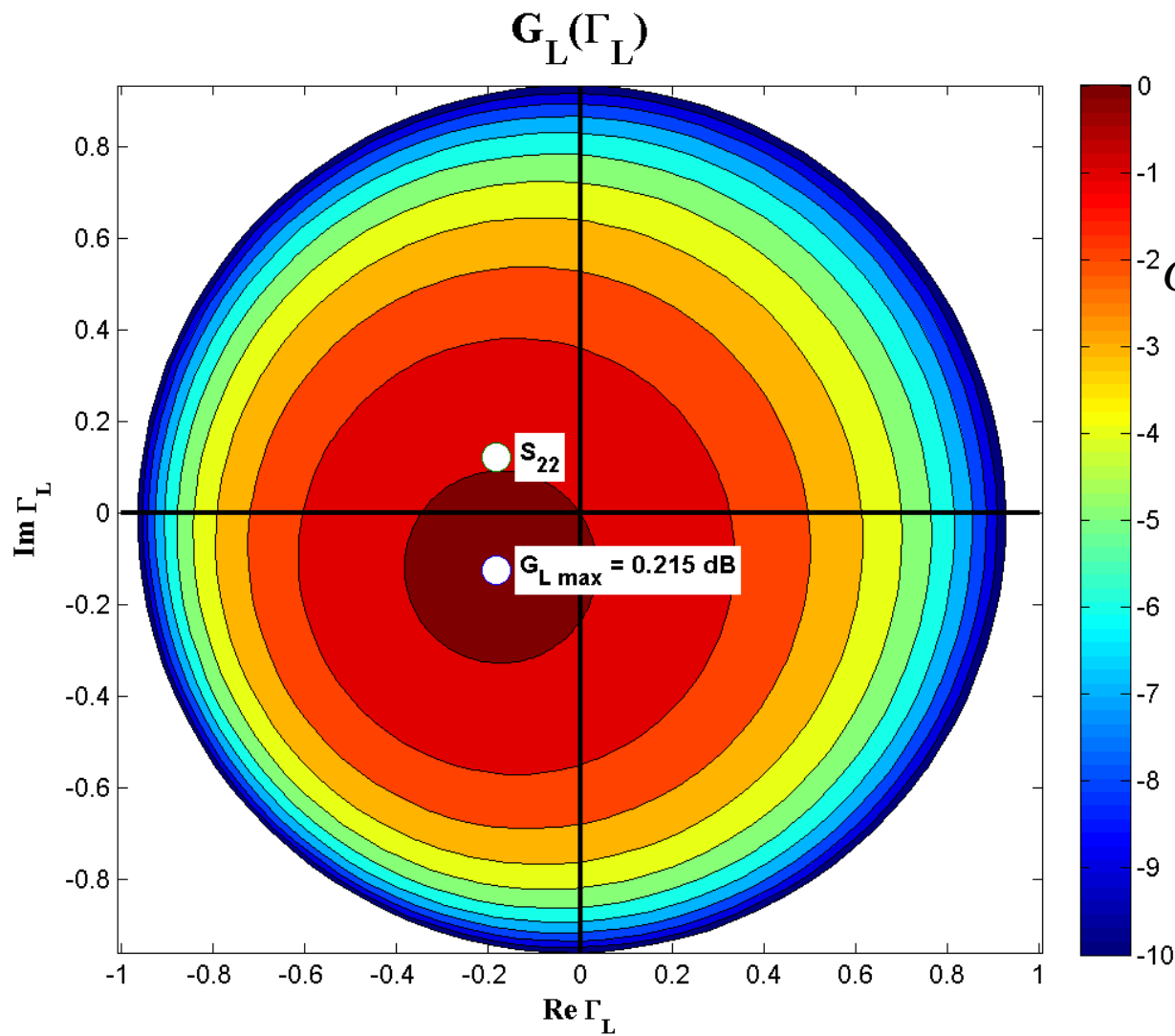
# $G_L(\Gamma_L)$ , constant value contours



$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

$$G_{L\max} = G_L \Big|_{\Gamma_L = S_{22}^*}$$

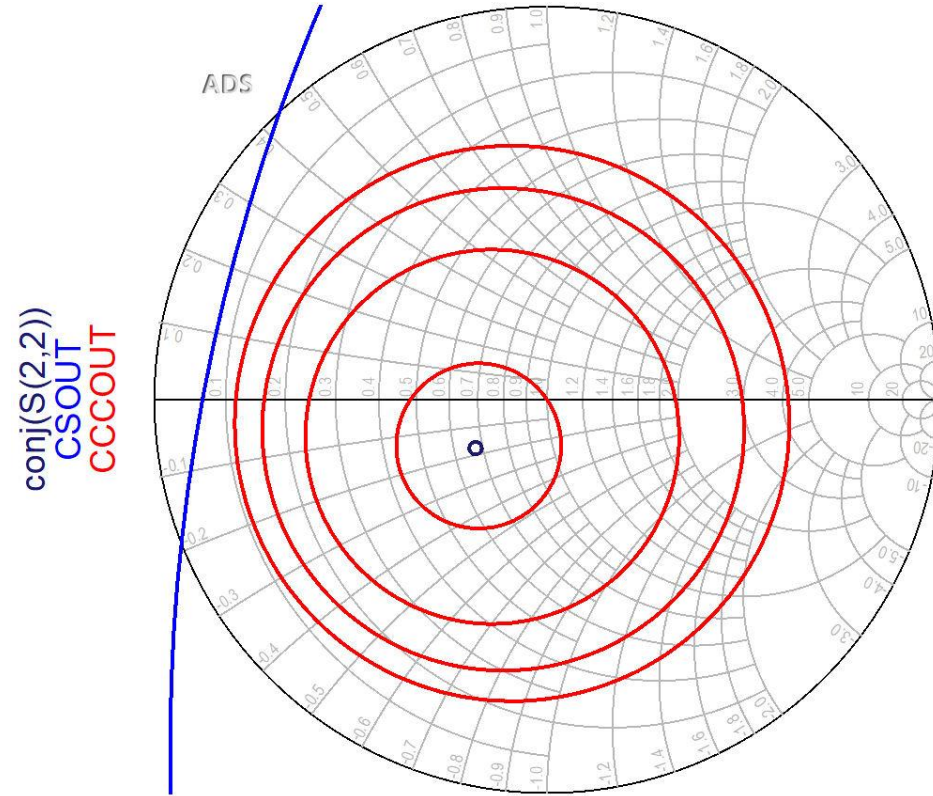
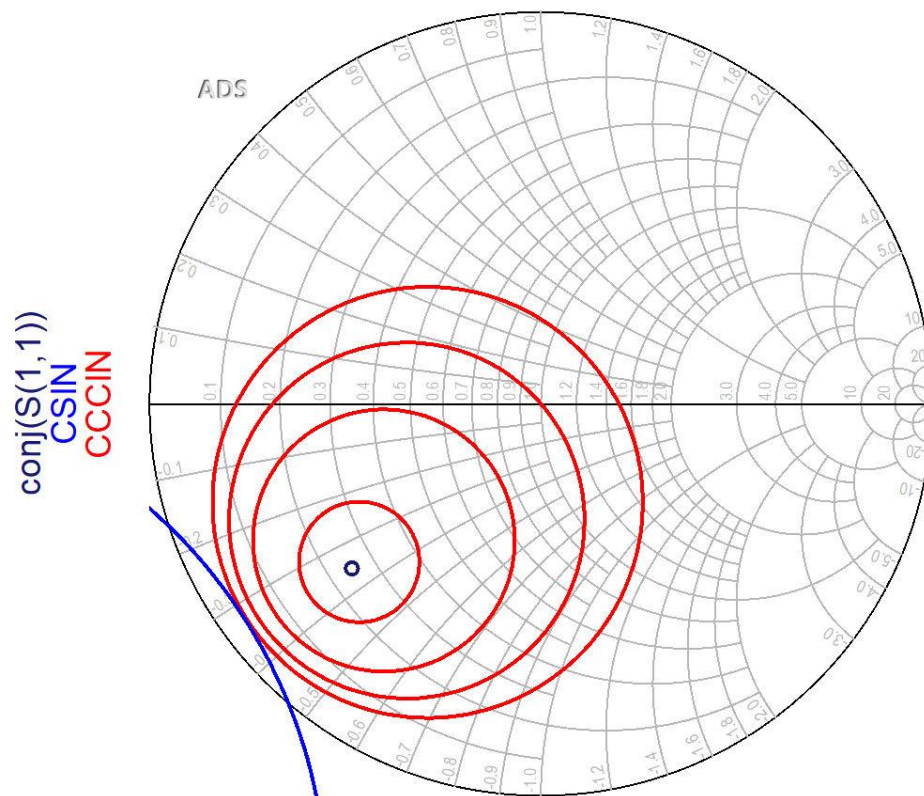
# $G_L[\text{dB}](\Gamma_L)$ , constant value contours



$$G_L[\text{dB}] = 10 \cdot \log \left( \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2} \right)$$

$$G_{L \text{ max}} = G_L \Big|_{\Gamma_L = S_{22}^*}$$

# ADS



- Circles are plotted for requested values (**in dB!**)
- It is useful to compute  $G_{S_{\max}}$  and  $G_{L_{\max}}$  before
  - in order to request relevant circles

# Design for Specified Gain

- We compute  $G_o$ ,  $G_{S_{max}}$ ,  $G_{L_{max}}$
- To obtain the design gain we **choose** supplemental gain needed (supplemental to constant  $G_o$ )
  - we account for the deviation that might arise from the unilateral assumption (using unilateral figure of merit U)

$$G_{design}[dB] = G_{S\_design}[dB] + G_o[dB] + G_{L\_design}[dB]$$

- We plot the circles for design (chosen) values  $G_{S\_design}$ ,  $G_{L\_design}$
- We design input and output matching circuits which move the reflection coefficient **on** or **inside** the design circles (depending on specific application requirements)

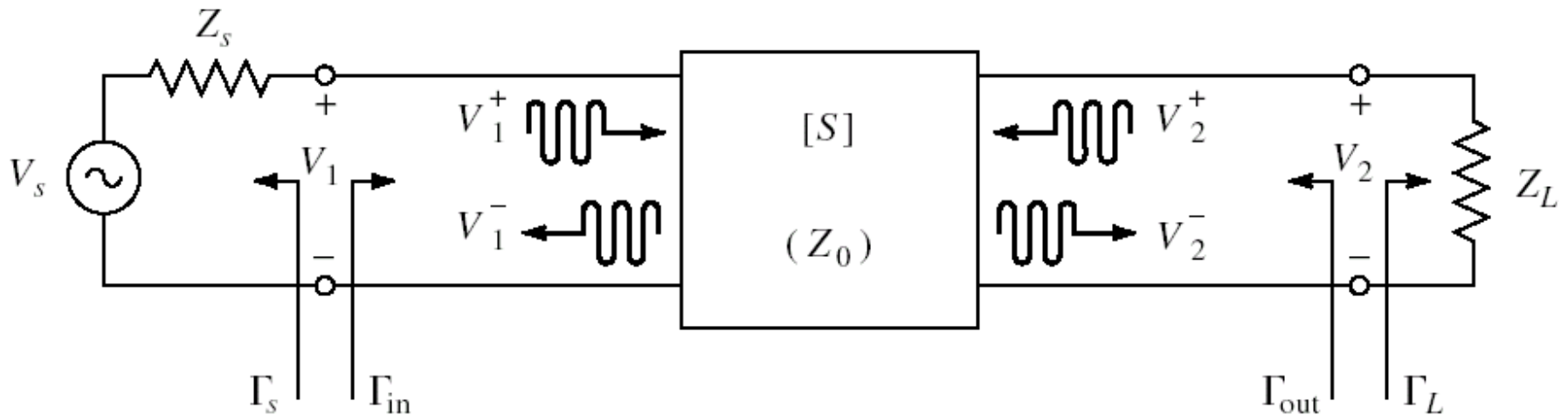


Low-Noise Amplifier Design

# Microwave Amplifiers

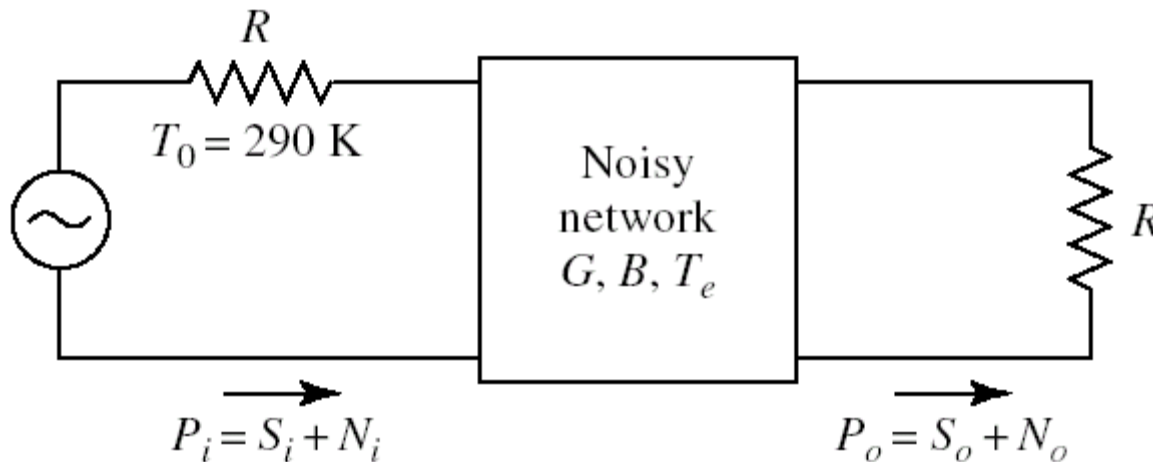
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# Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - power gain
  - **noise** (sometimes – small signals)
  - linearity (sometimes – large signals)

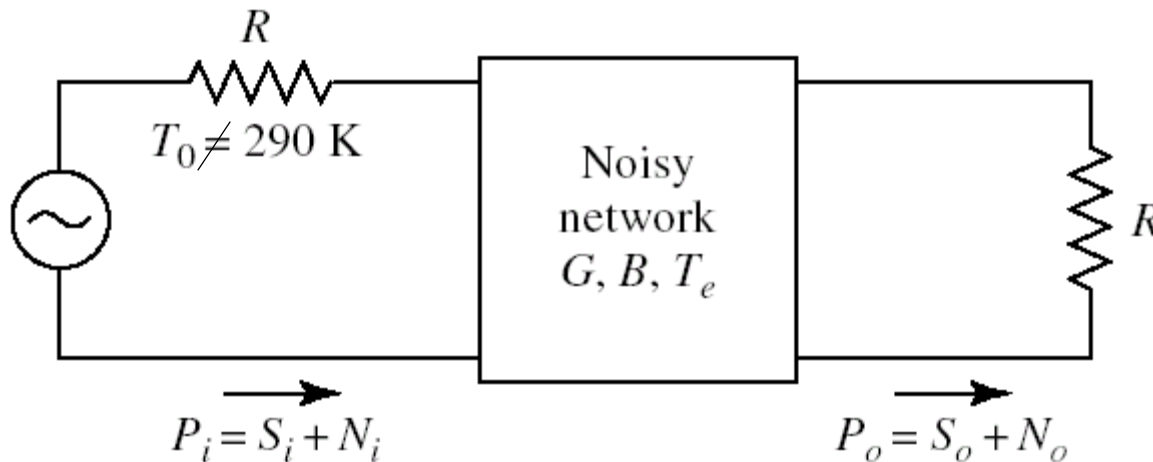
# Noise Figure F



- The noise figure  $F$ , is a measure of the reduction in signal-to-noise ratio between the input and output of a device, when (by definition) the input noise power is assumed to be the noise power resulting from a matched resistor at  $T_0 = 290\text{ K}$  (reference noise conditions)

$$F = \left. \frac{S_i/N_i}{S_o/N_o} \right|_{T_0=290K}$$

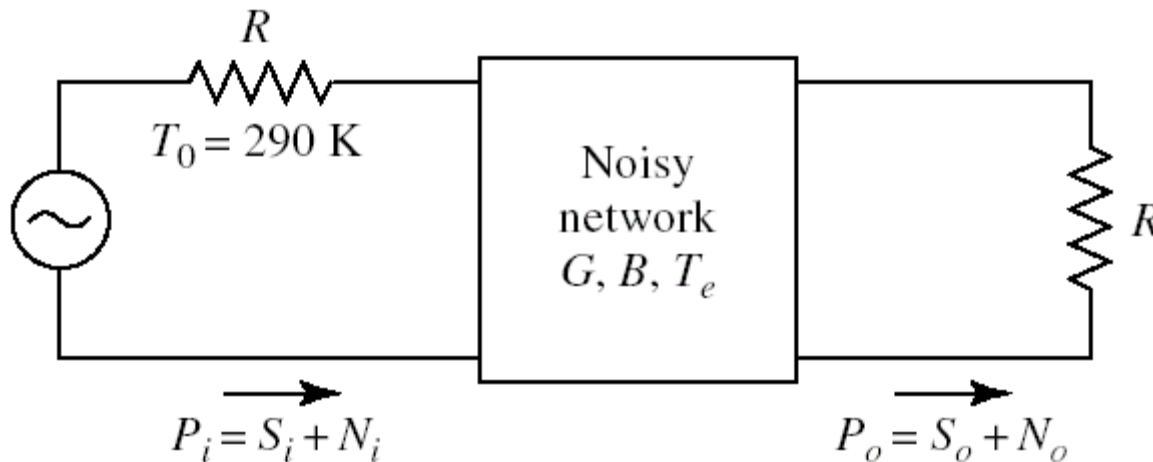
# Noise Figure F



- The noise figure  $F$ , **is not** directly a measure of the reduction in signal-to-noise ratio between the input and output of a device, when the input noise power is different from that of the reference noise conditions

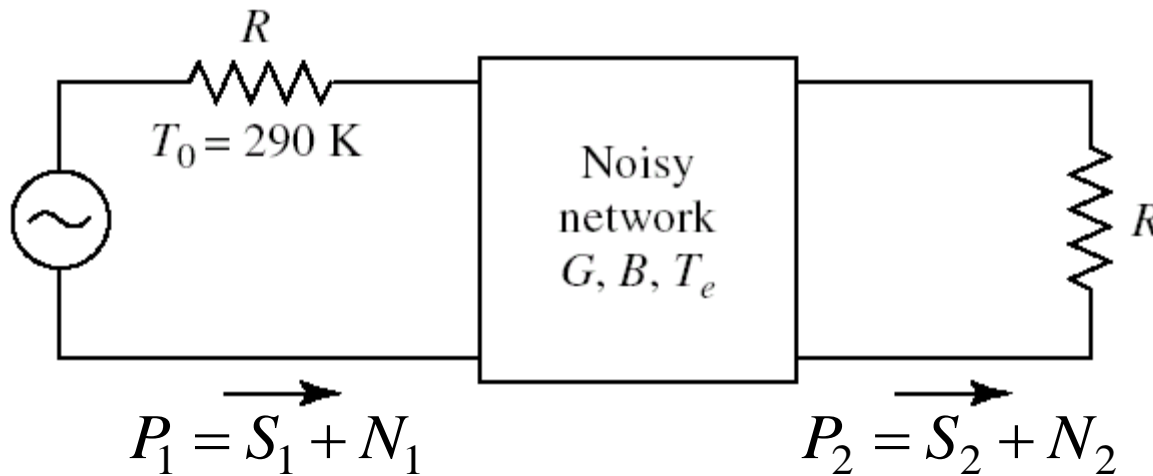
$$F \neq \left. \frac{S_i/N_i}{S_o/N_o} \right|_{T_0 \neq 290 \text{ K}}$$

# Noise Figure F



- In general, the output noise power consists of two elements:
  - the input noise power amplified or attenuated by the device (for example amplified with the power gain  $G$  applied also to the desired signal)
  - a noise power generated internally by the network if the network is noisy (this power **does not** depend on the input noise power)

# Noise Figure F



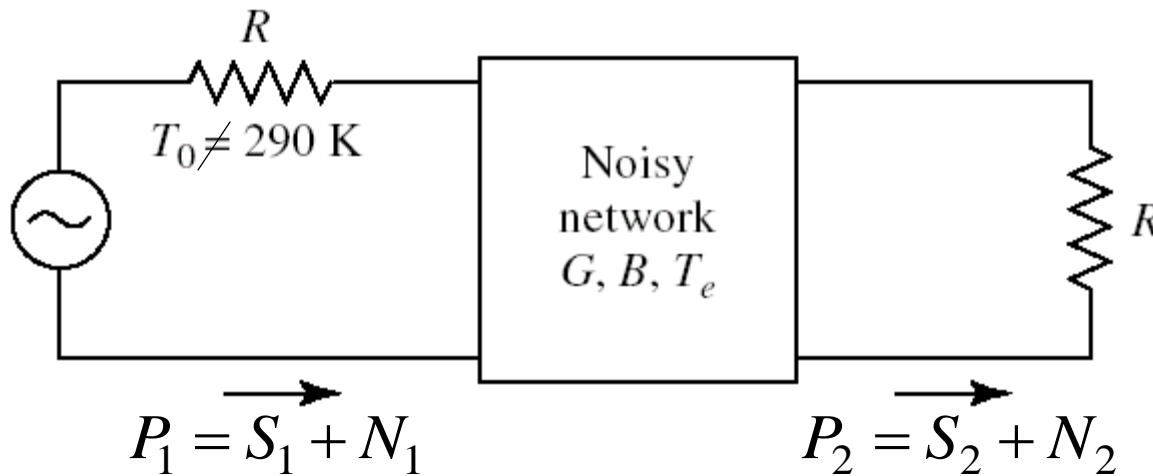
- Estimation of the internally generated noise power can be done using the Noise Figure  $F$  definition:

$$F = \left. \frac{S_1/N_1}{S_2/N_2} \right|_{T_0=290\text{K}, N_1=N_0}$$

$$N_2 = F \cdot N_0 \cdot \frac{S_2}{S_1} = F \cdot N_0 \cdot G$$

$$N_2 = N_0 \cdot G + (F - 1) \cdot N_0 \cdot G$$

# Noise Figure F

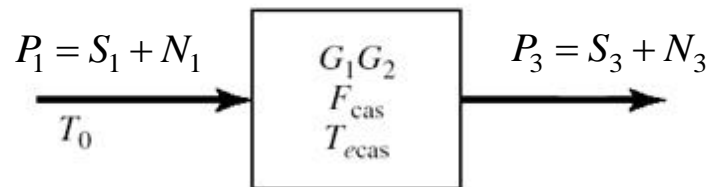
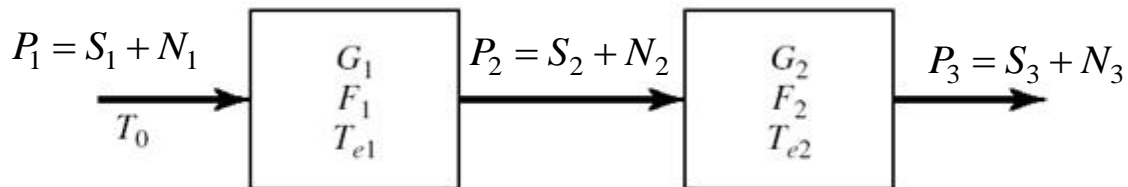


- We identify the two terms:
  - amplified input noise
  - internally generated noise
- When the input noise does not correspond to reference noise conditions ( $N_1 \neq N_0$ )
  - the internally generated noise does not change

$$N_2 = N_0 \cdot G + (F - 1) \cdot N_0 \cdot G$$

$$N_2 = N_1 \cdot G + (F - 1) \cdot N_0 \cdot G$$

# Noise figure of a cascaded system



$$N_2 = N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1$$

$$G_{cas} = G_1 \cdot G_2$$

$$N_3 = N_2 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$

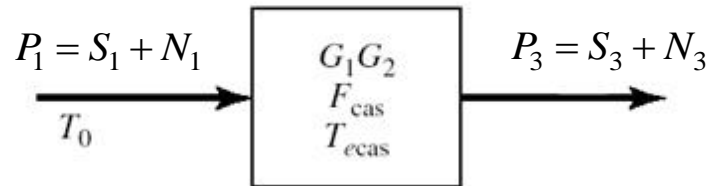
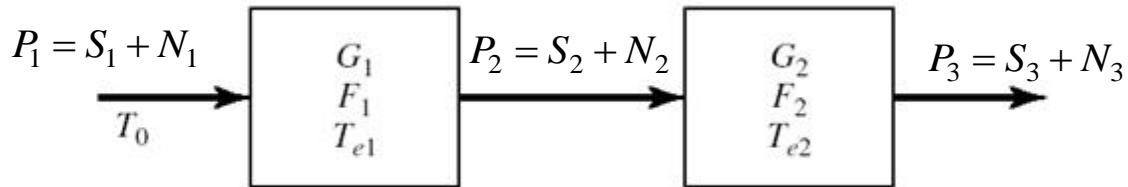


$$N_3 = [N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1] \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$



# Noise figure of a cascaded system



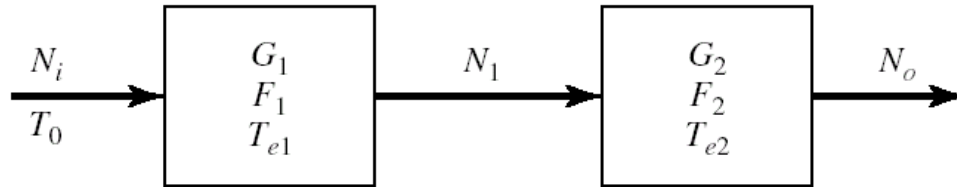
$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$G_{cas} = G_1 \cdot G_2 \quad N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$

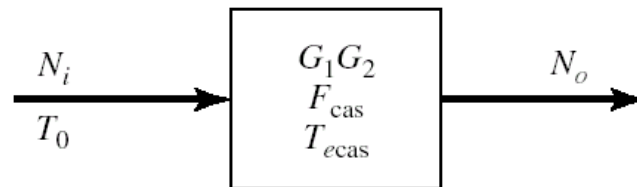
$$(F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2 = (F_{cas} - 1) \cdot N_0 \cdot G_1 \cdot G_2$$

$$F_{cas} = F_1 + \frac{1}{G_1} (F_2 - 1)$$

# Noise figure of a cascaded system



(a)



(b)

$$G_{cas} = G_1 \cdot G_2 \qquad F_{cas} = F_1 + \frac{1}{G_1} (F_2 - 1)$$

- Friis Formula (!linear scale)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \dots$$

# Friis Formula (noise)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \dots$$

- Friis Formula shows that:
  - the overall noise figure of a cascaded system is largely determined by the noise characteristics of the first stage
  - the noise introduced by the following stages is reduced:
    - -1
    - division by G (usually  $G > 1$ )

# Friis Formula (noise)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \dots$$

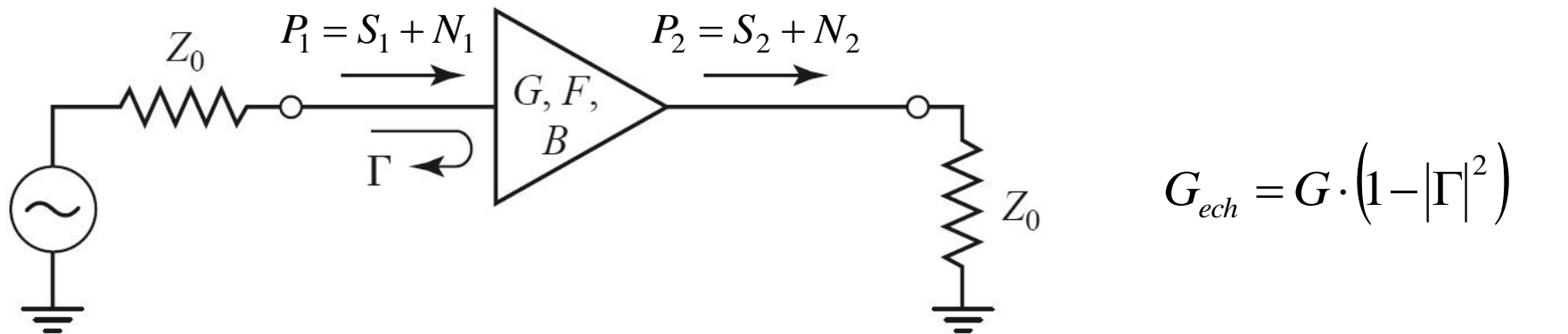
- Effects of Friis Formula:
- in multi stage amplifiers:
  - it's essential that the first stage is as noiseless as possible even if that means sacrificing power gain
  - the following stages can be optimized for power gain
- in single stage amplifiers:
  - in the input matching circuit it's important to have noiseless elements (pure reactance, lossless lines)
  - output matching circuit has less influence on the noise (noise generated at this level appears when the desired signal has already been amplified by the transistor)

$$V_{n(ef)} = \sqrt{4kTBR}$$

$$P_n = kTB$$

# Noise Figure of a Mismatched Amplifier

- An input mismatched amplifier ( $\Gamma \neq 0$ )



$$N_2 = N_1 \cdot G \cdot (1 - |\Gamma|^2) + (F - 1) \cdot N_0 \cdot G = N_1 \cdot G \cdot (1 - |\Gamma|^2) + \frac{F - 1}{1 - |\Gamma|^2} \cdot N_0 \cdot G \cdot (1 - |\Gamma|^2)$$

$$N_2 = N_1 \cdot G_{ech} + (F_{ech} - 1) \cdot N_0 \cdot G_{ech} \quad F_{ech} = 1 + \frac{F - 1}{1 - |\Gamma|^2} \geq F$$

- Good noise figure **requires** good impedance matching

# Example

- ATF-34143 at  $V_{ds}=3V$   $I_d=20mA$ .
- @5GHz
  - $S_{11} = 0.64 \angle 139^\circ$
  - $S_{12} = 0.119 \angle -21^\circ$
  - $S_{21} = 3.165 \angle 16^\circ$
  - $S_{22} = 0.22 \angle 146^\circ$
  - $F_{min} = 0.54$  (tipic [dB])
  - $\Gamma_{opt} = 0.45 \angle 174^\circ$
  - $r_n = 0.03$

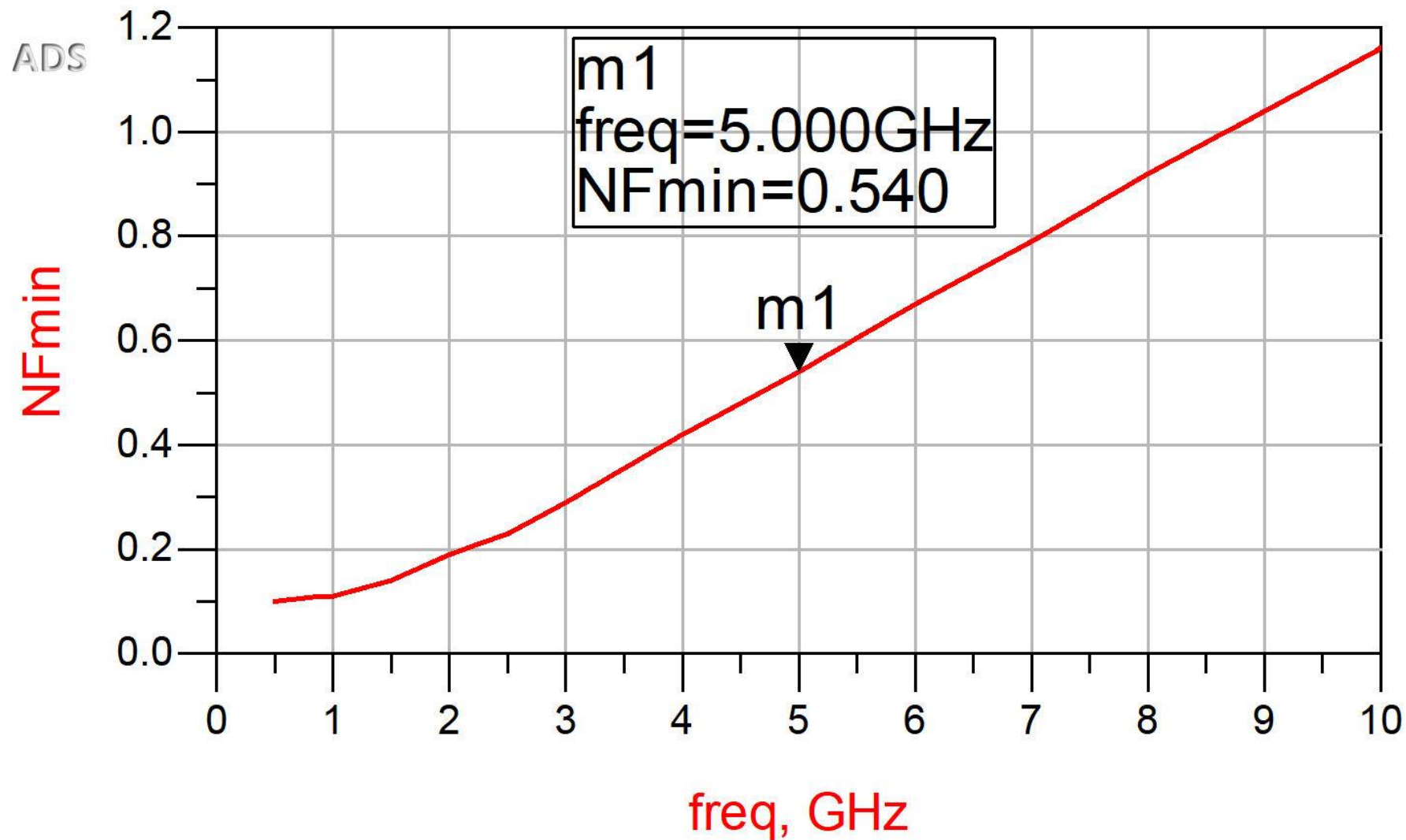
```
IATF-34143
IS-PARAMETERS at Vds=3V Id=20mA. LAST UPDATED 01-29-99

# ghz s ma r 50

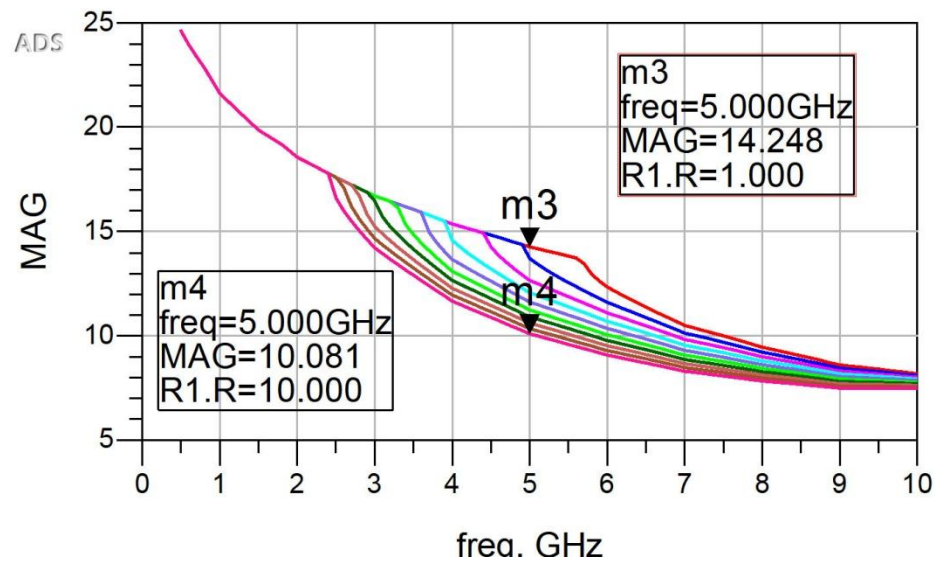
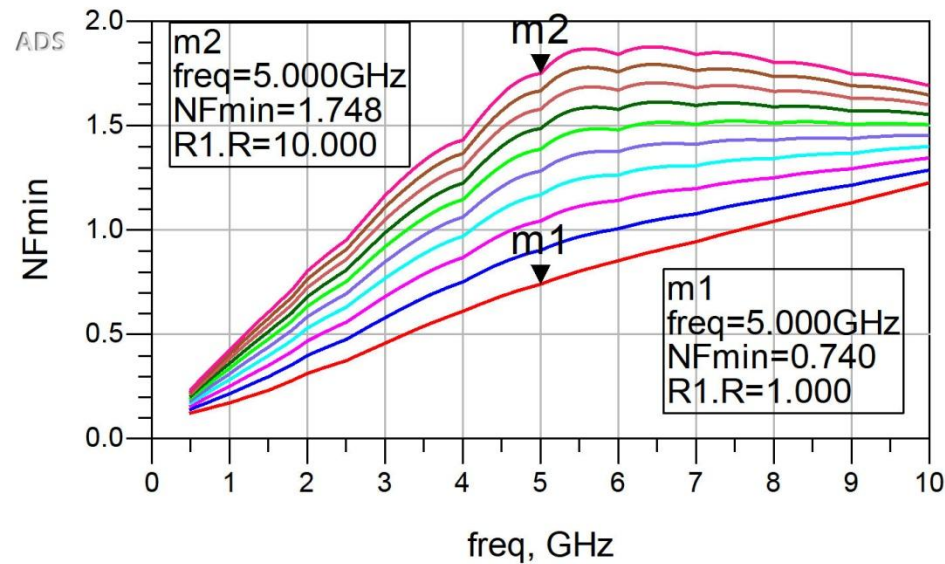
2.0 0.75 -126 6.306 90 0.088 23 0.26 -120
2.5 0.72 -145 5.438 75 0.095 15 0.25 -140
3.0 0.69 -162 4.762 62 0.102 7 0.23 -156
4.0 0.65 166 3.806 38 0.111 -8 0.22 174
5.0 0.64 139 3.165 16 0.119 -21 0.22 146
6.0 0.65 114 2.706 -5 0.125 -35 0.23 118
7.0 0.66 89 2.326 -27 0.129 -49 0.25 91
8.0 0.69 67 2.017 -47 0.133 -62 0.29 67
9.0 0.72 48 1.758 -66 0.135 -75 0.34 46

!FREQ Fopt GAMMA OPT RN/Zo
!GHZ dB MAG ANG -
2.0 0.19 0.71 66 0.09
2.5 0.23 0.65 83 0.07
3.0 0.29 0.59 102 0.06
4.0 0.42 0.51 138 0.03
5.0 0.54 0.45 174 0.03
6.0 0.67 0.42 -151 0.05
7.0 0.79 0.42 -118 0.10
8.0 0.92 0.45 -88 0.18
9.0 1.04 0.51 -63 0.30
10.0 1.16 0.61 -43 0.46
```

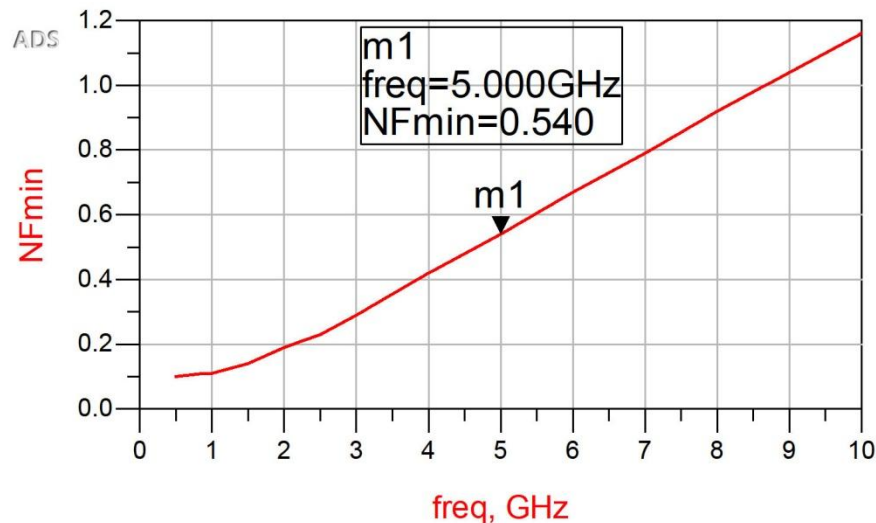
# Example



# Stabilization, input series resistor

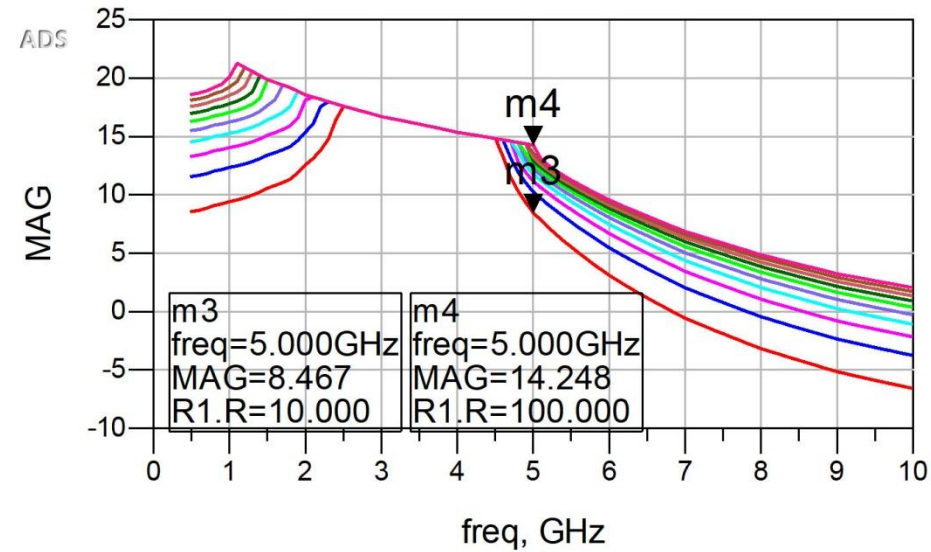
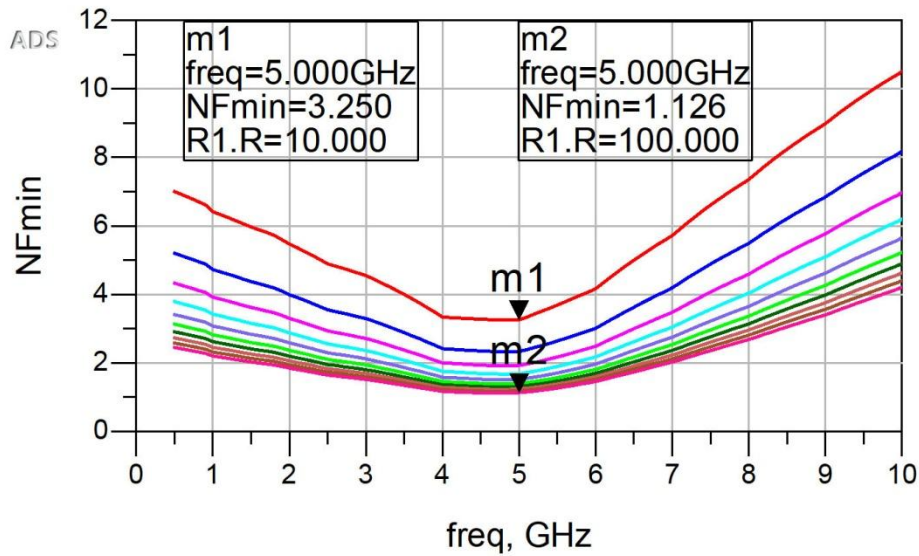


$$R_{SS} = 1 \div 10 \Omega$$

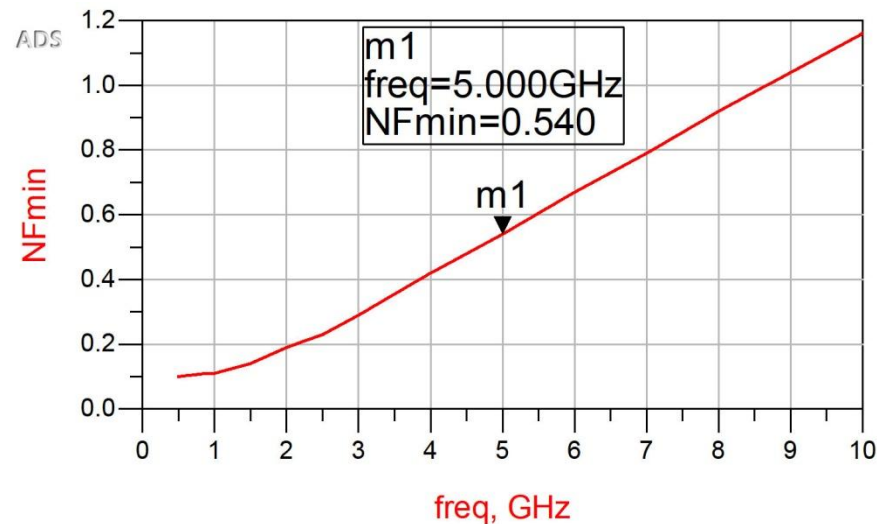




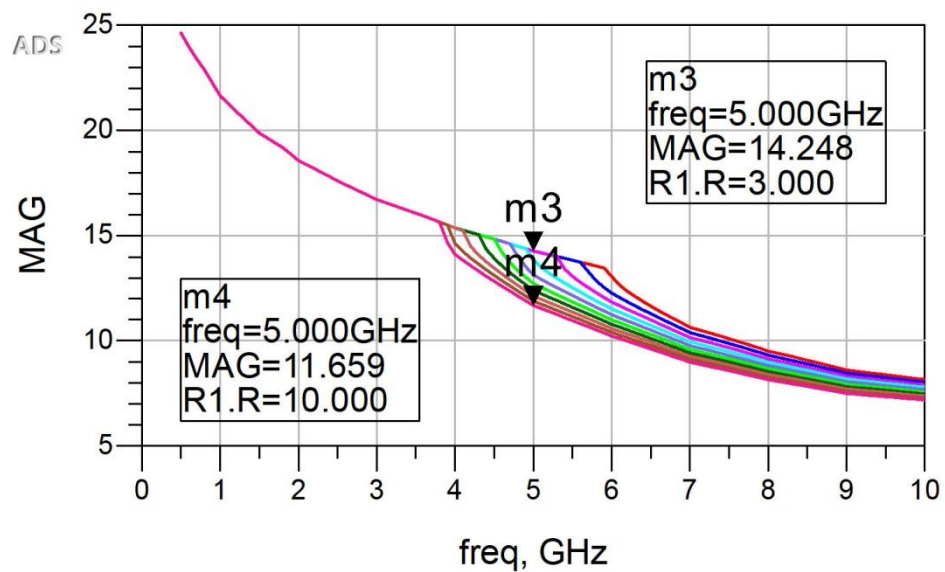
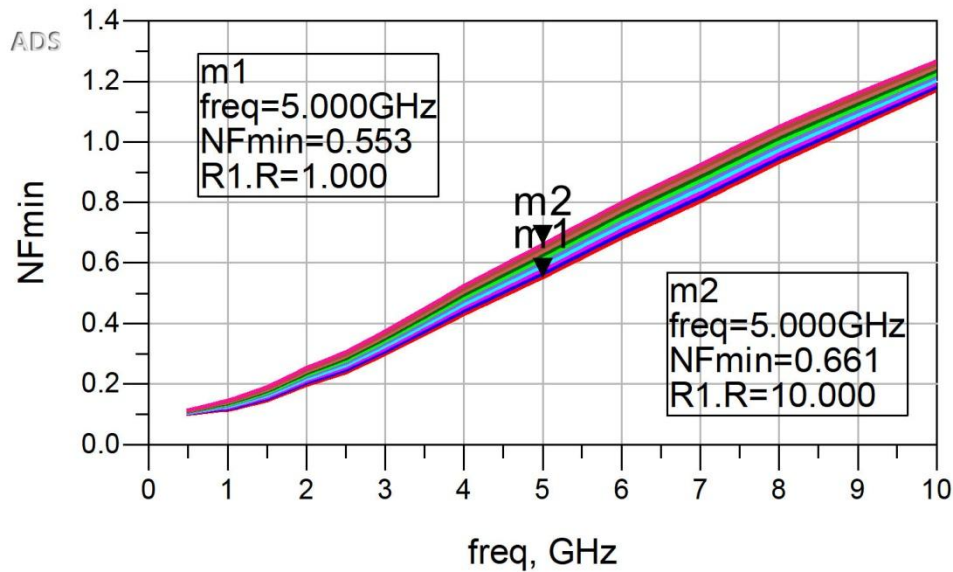
# Stabilization, input shunt resistor



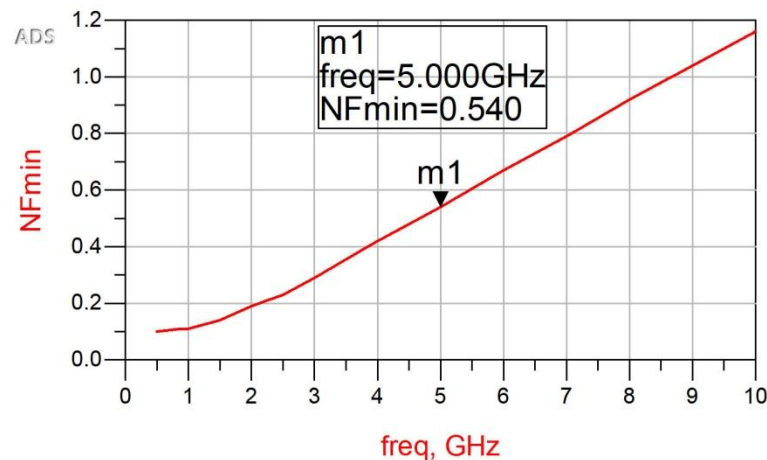
$$R_{PS} = 10 \div 100 \Omega$$



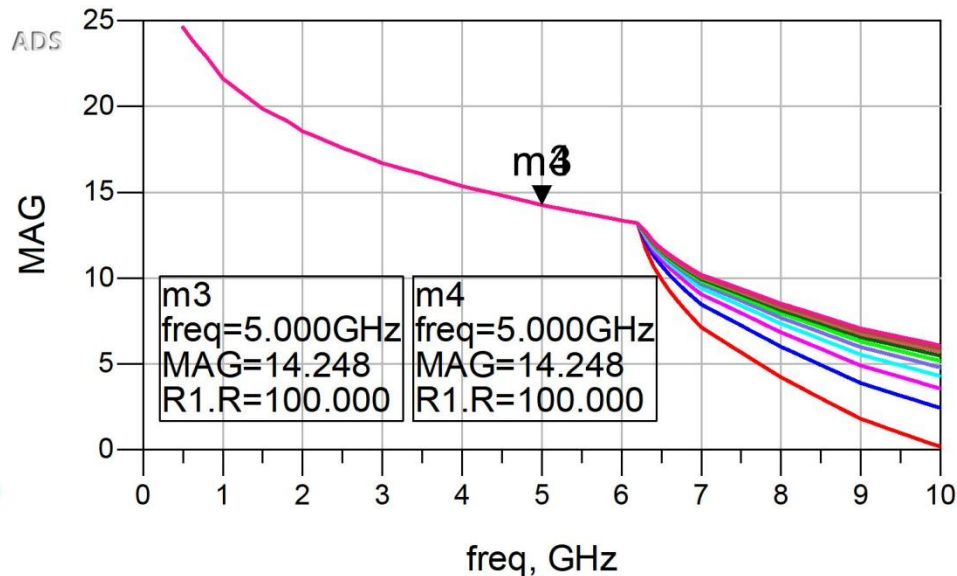
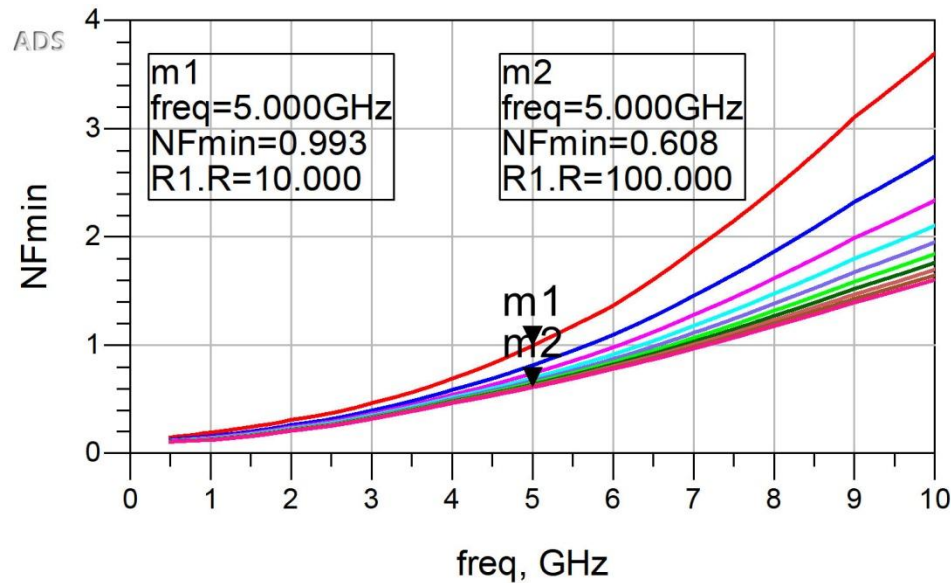
# Stabilization, output series resistor



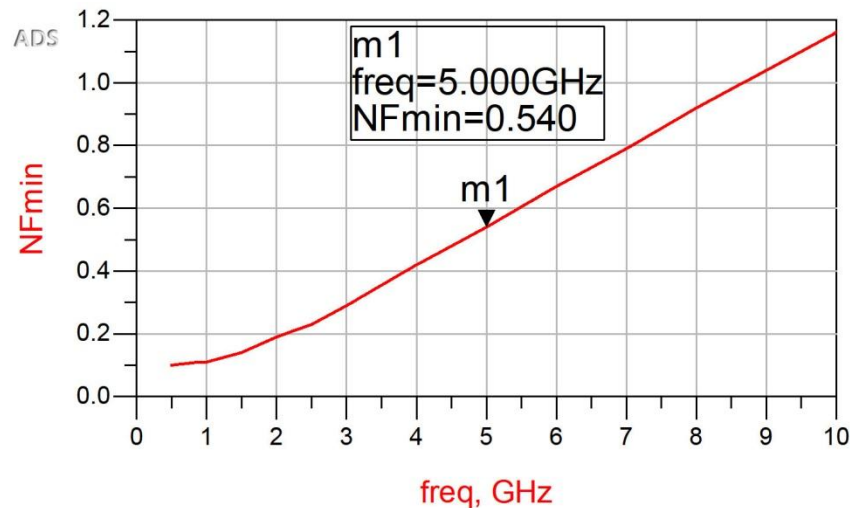
$$R_{SL} = 1 \div 10 \Omega$$



# Stabilization, output shunt resistor



$$R_{PL} = 10 \div 100 \Omega$$



# Noise figure of a two-port amplifier

- 3 noise parameters (2 reals + 1 complex):

$$F_{\min}, r_n = \frac{R_N}{Z_0}, \Gamma_{opt}$$

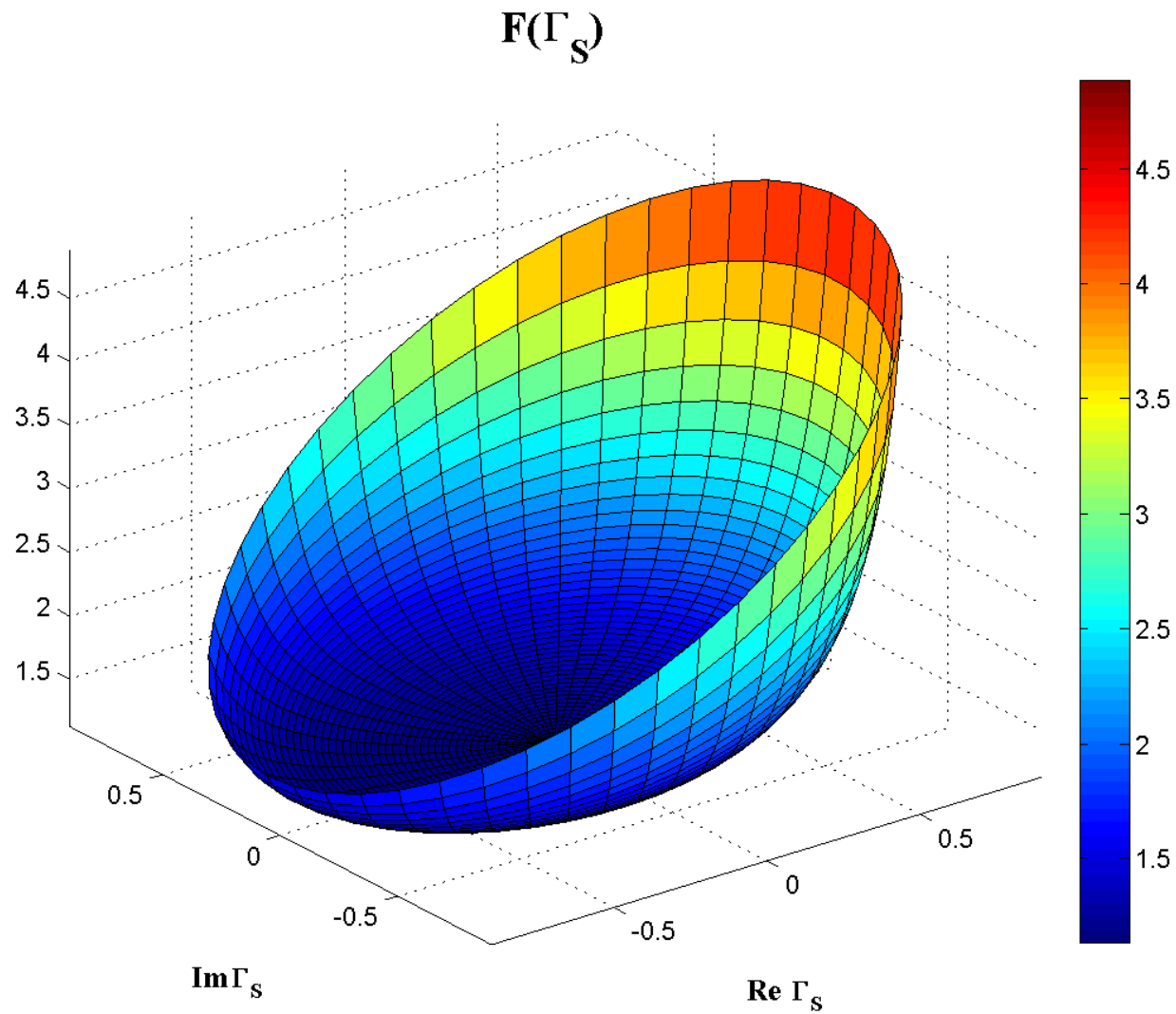
$$F = F_{\min} + \frac{R_N}{G_S} \cdot |Y_S - Y_{opt}|^2 \quad Y_S = \frac{1}{Z_0} \cdot \frac{1 - \Gamma_S}{1 + \Gamma_S} \quad Y_{opt} = \frac{1}{Z_0} \cdot \frac{1 - \Gamma_{opt}}{1 + \Gamma_{opt}}$$

$$F = F_{\min} + 4 \cdot r_n \cdot \frac{|\Gamma_S - \Gamma_{opt}|^2}{(1 - |\Gamma_S|^2) \cdot |1 + \Gamma_{opt}|^2}$$

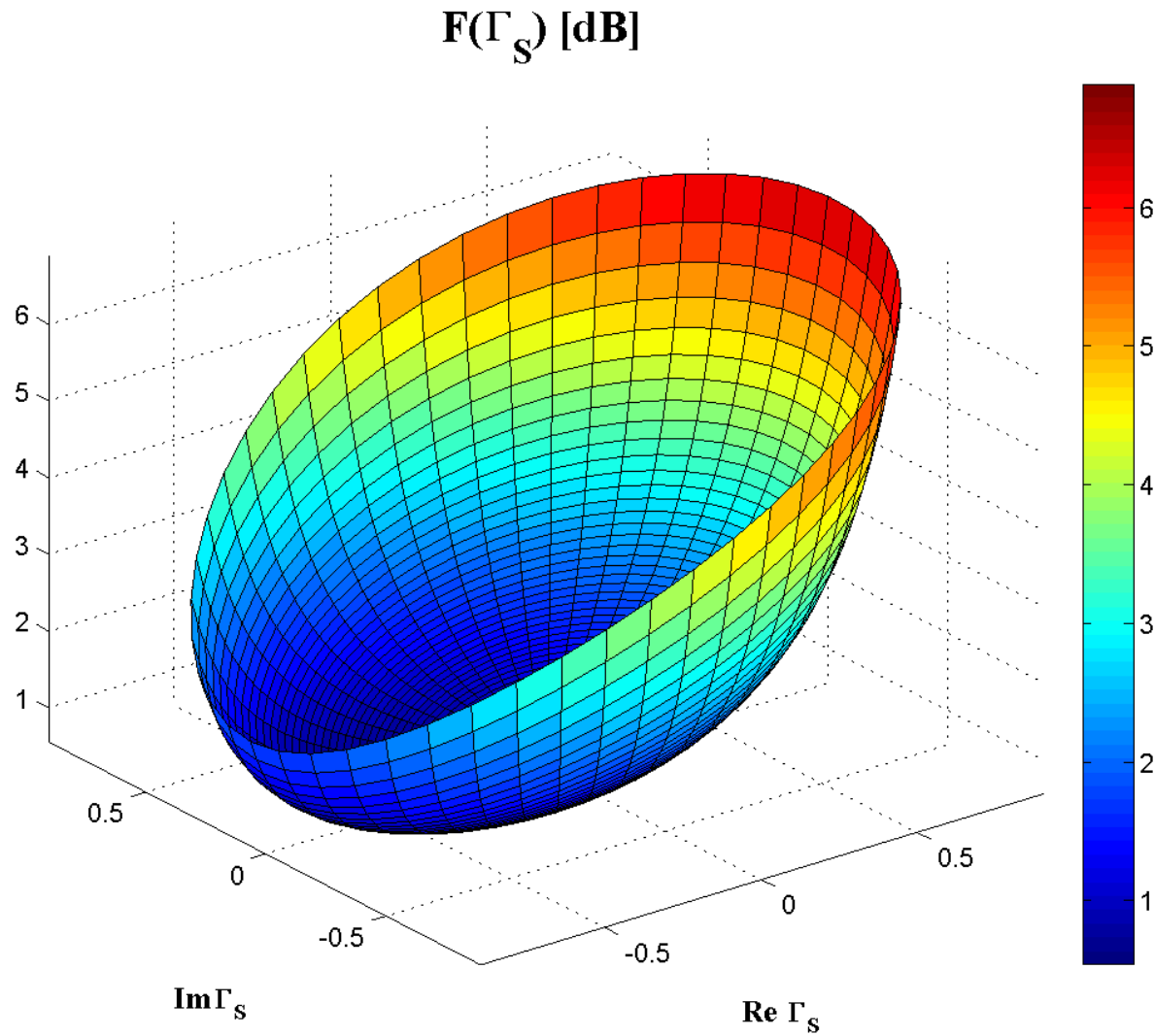
- $\Gamma_{opt}$  optimum source reflection coefficient that results in minimum noise figure

$$\Gamma_S = \Gamma_{opt} \Rightarrow F = F_{\min}$$

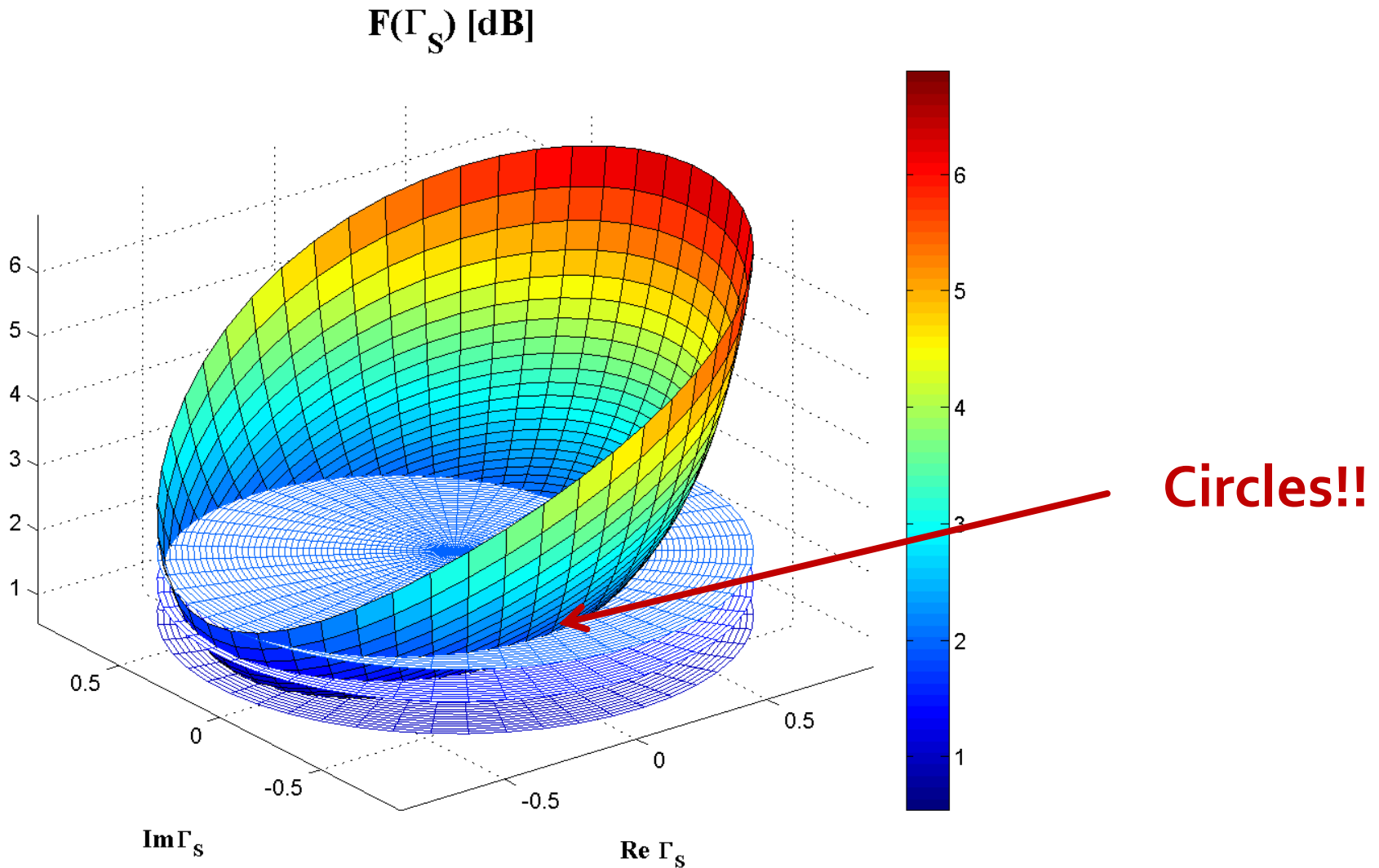
# $F(\Gamma_S)$



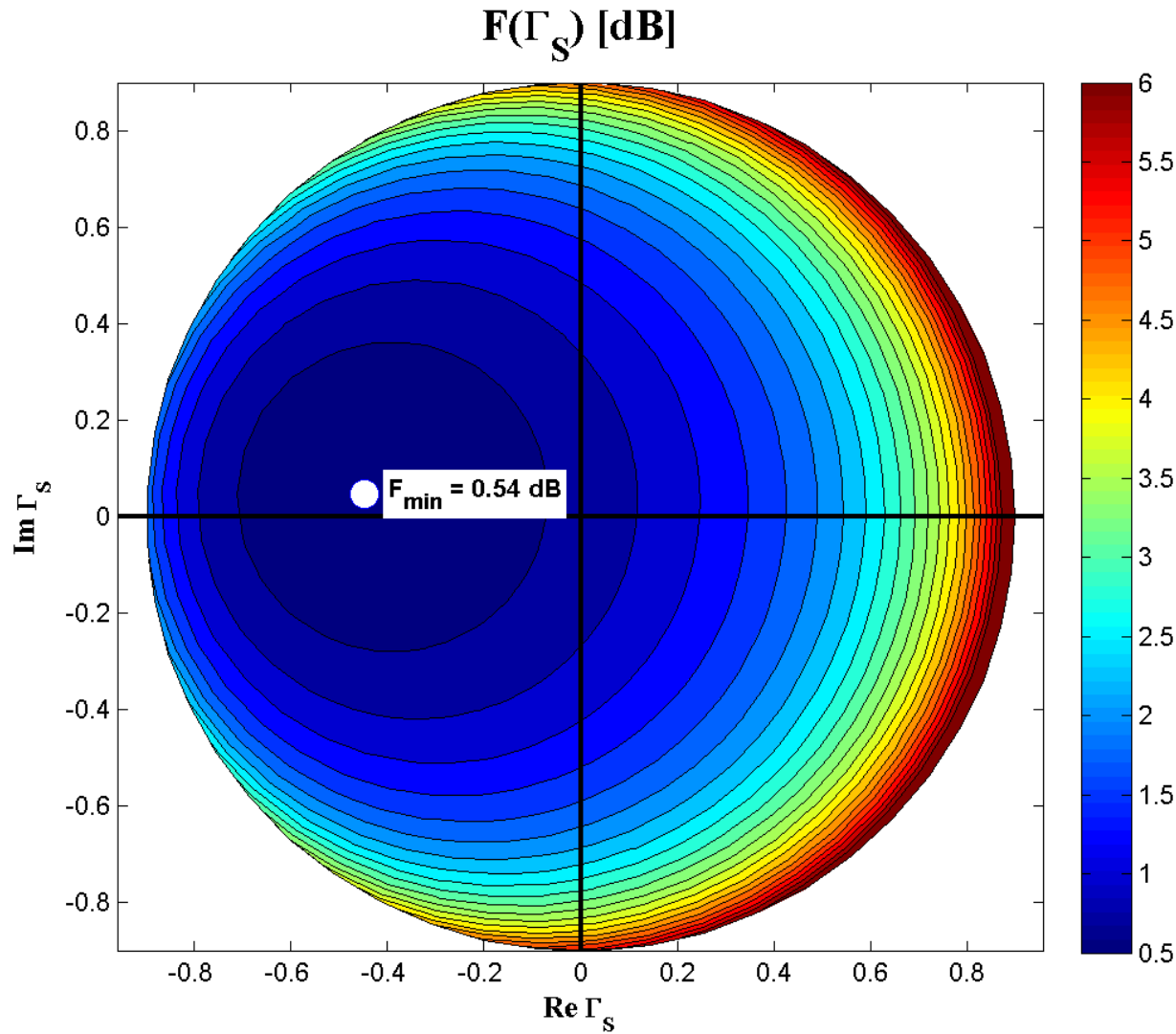
# $F[\text{dB}](\Gamma_S)$



# $F[\text{dB}](\Gamma_s)$ , constant value contours



# $G_S[\text{dB}](\Gamma_S)$ , constant value contours



$$\Gamma_{\text{opt}} = 0.45 \angle 174^\circ$$



# Circles of constant noise figure

- We define N (noise figure parameter)
  - N constant for F constant

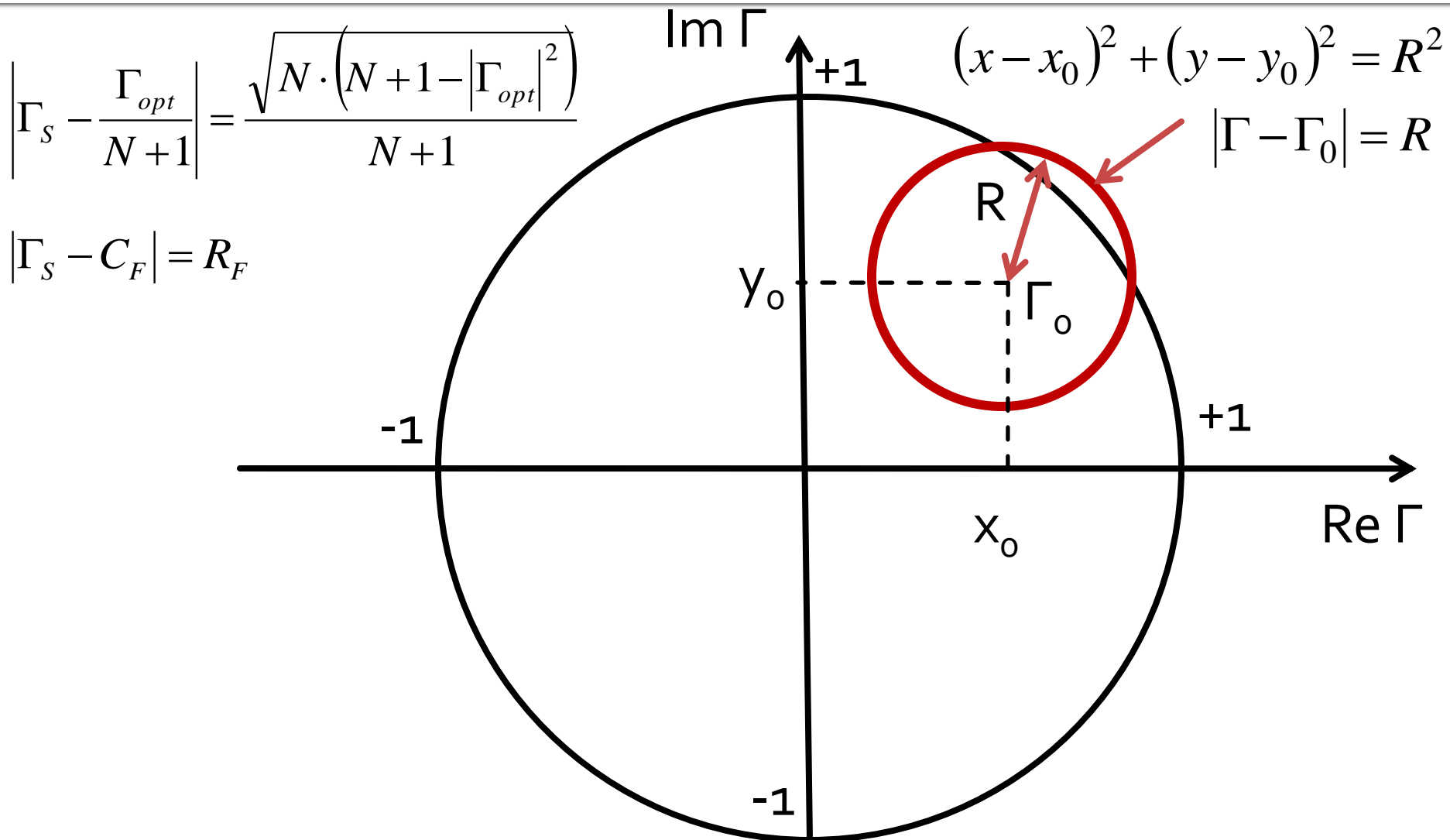
$$N = \frac{|\Gamma_S - \Gamma_{opt}|^2}{1 - |\Gamma_S|^2} = \frac{F - F_{min}}{4 \cdot r_n} \cdot |1 + \Gamma_{opt}|^2$$

$$(\Gamma_S - \Gamma_{opt}) \cdot (\Gamma_S^* - \Gamma_{opt}^*) = N \cdot (1 - |\Gamma_S|^2)$$

$$\Gamma_S \cdot \Gamma_S^* + N \cdot |\Gamma_S|^2 - (\Gamma_S \cdot \Gamma_{opt}^* - \Gamma_S^* \cdot \Gamma_{opt}) + \Gamma_{opt} \cdot \Gamma_{opt}^* = N$$

$$\Gamma_S \cdot \Gamma_S^* - \frac{\Gamma_S \cdot \Gamma_{opt}^* - \Gamma_S^* \cdot \Gamma_{opt}}{N + 1} + \Gamma_{opt} \cdot \Gamma_{opt}^* = \frac{N - |\Gamma_{opt}|^2}{N + 1} + \frac{|\Gamma_{opt}|^2}{(N + 1)^2}$$

# Circles of constant noise figure

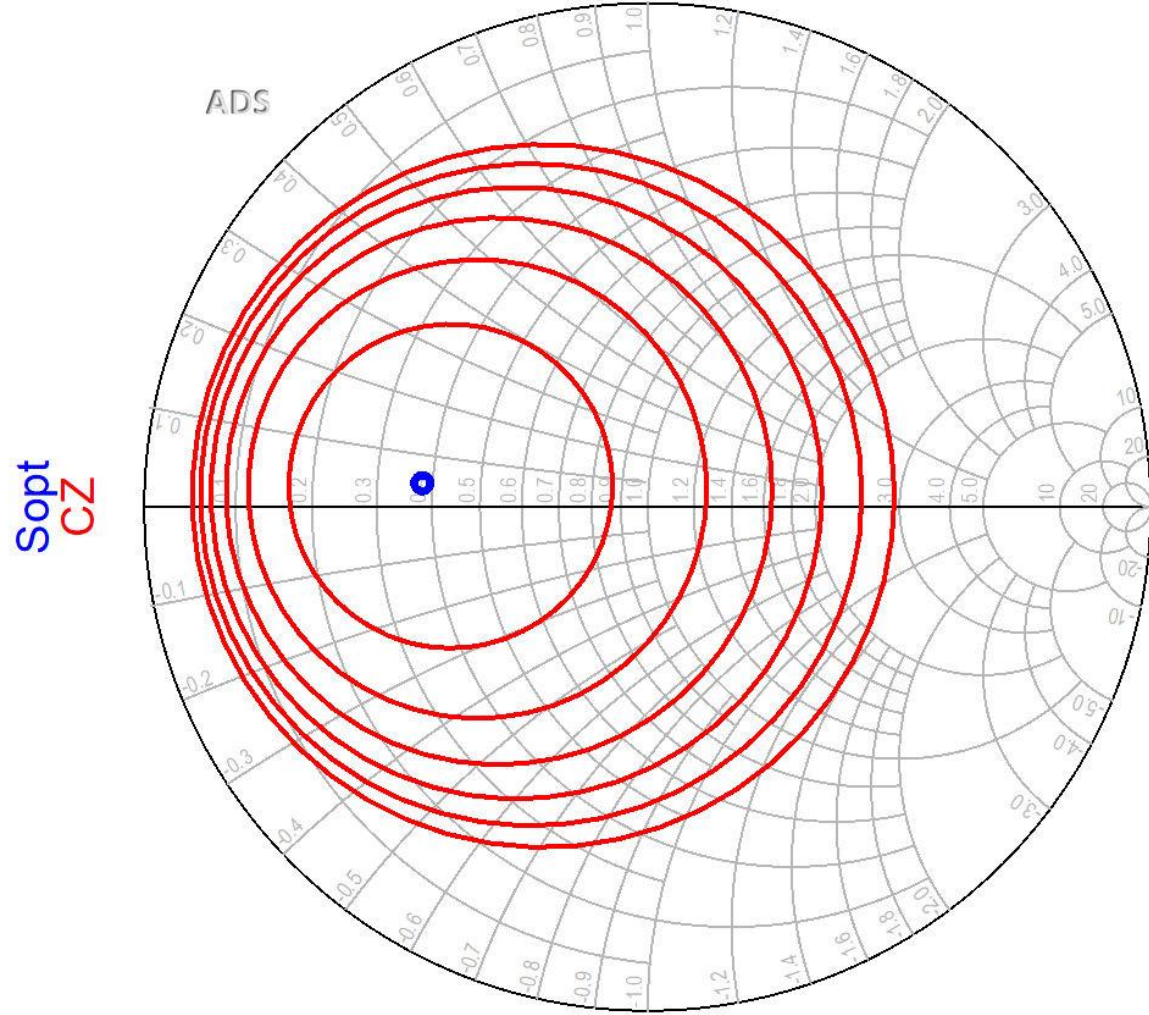


# Circles of constant noise figure

$$\left| \Gamma_S - \frac{\Gamma_{opt}}{N+1} \right| = \frac{\sqrt{N \cdot (N+1 - |\Gamma_{opt}|^2)}}{N+1} \quad |\Gamma_S - C_F| = R_F$$
$$C_F = \frac{\Gamma_{opt}}{N+1} \quad R_F = \frac{\sqrt{N \cdot (N+1 - |\Gamma_{opt}|^2)}}{N+1}$$

- The locus in the complex plane  $\Gamma_S$  of the points with constant noise figure is a circle
- **Interpretation:** Any reflection coefficient  $\Gamma_S$  which plotted in the complex plane lies **on** the circle drawn for  $F_{circle}$  will lead to a noise factor  $F = F_{circle}$ 
  - Any reflection coefficient  $\Gamma_S$  plotted **outside** this circle will lead to a noise factor  $F > F_{circle}$
  - Any reflection coefficient  $\Gamma_S$  plotted **inside** this circle will lead to a noise factor  $F < F_{circle}$

# ADS

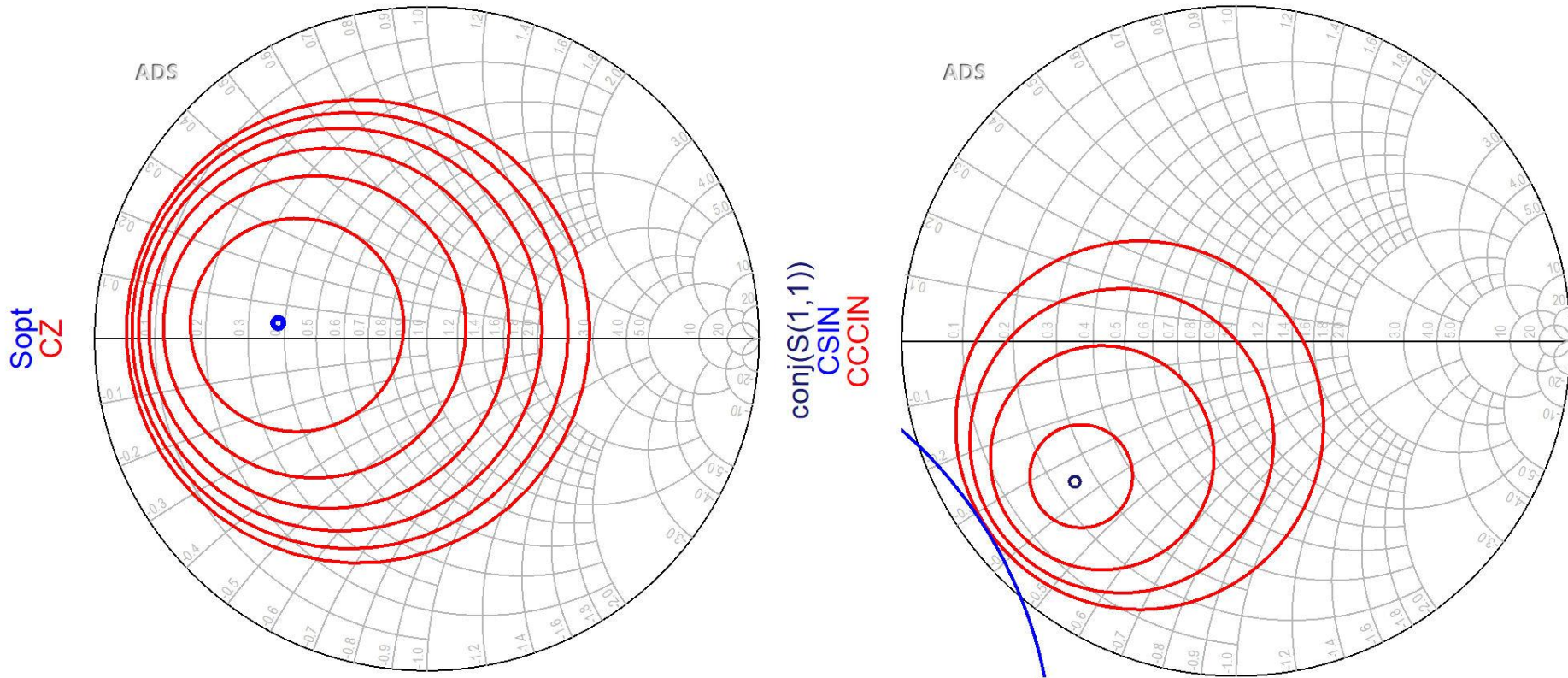


# Circles of constant noise figure

- The noise internally generated by the transistor depends **only** by the input matching circuit
- A minimum noise figure is possible ( $NF_{\min}$  – a datasheet parameter for the transistor)
- If we design a low noise amplifier (**LNA**) the usual design technique is as follows:
  - design of the input matching circuit solely (largely) for noise optimization
  - design of output matching circuit for gain compensation/optimization (if lossy circuits are used the output matching circuit noise can be added but the transistor noise is not influenced)

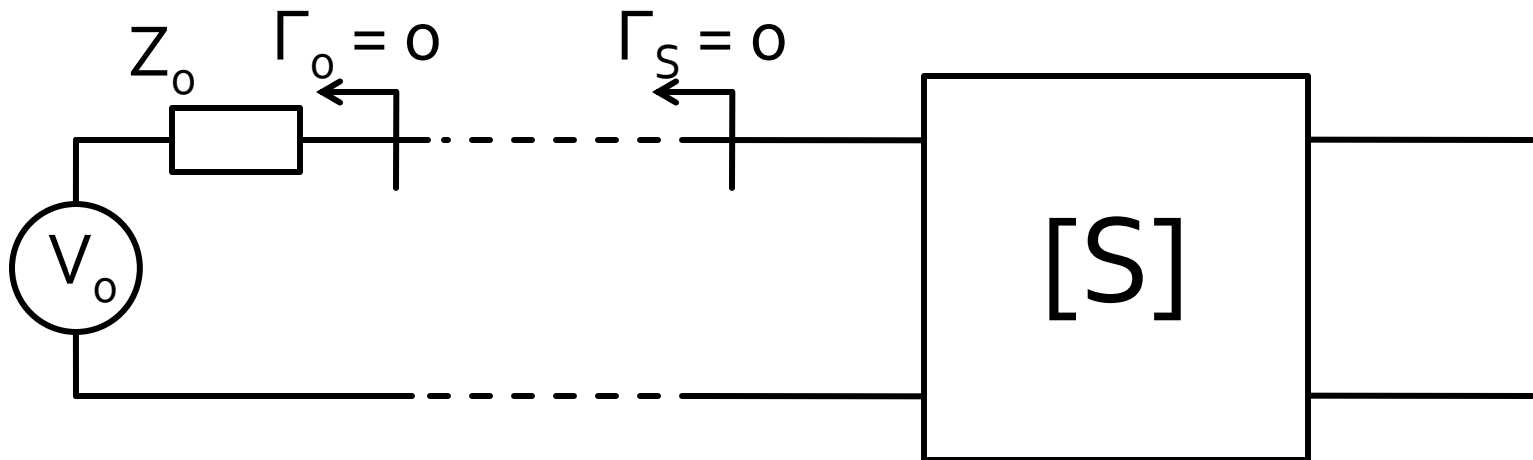
# LNA – Low Noise Amplifier

- Usually a transistor suitable for implementing an LNA at a certain frequency will have input gain circles and noise circles in the same area for  $\Gamma_S$



# Matching – 1

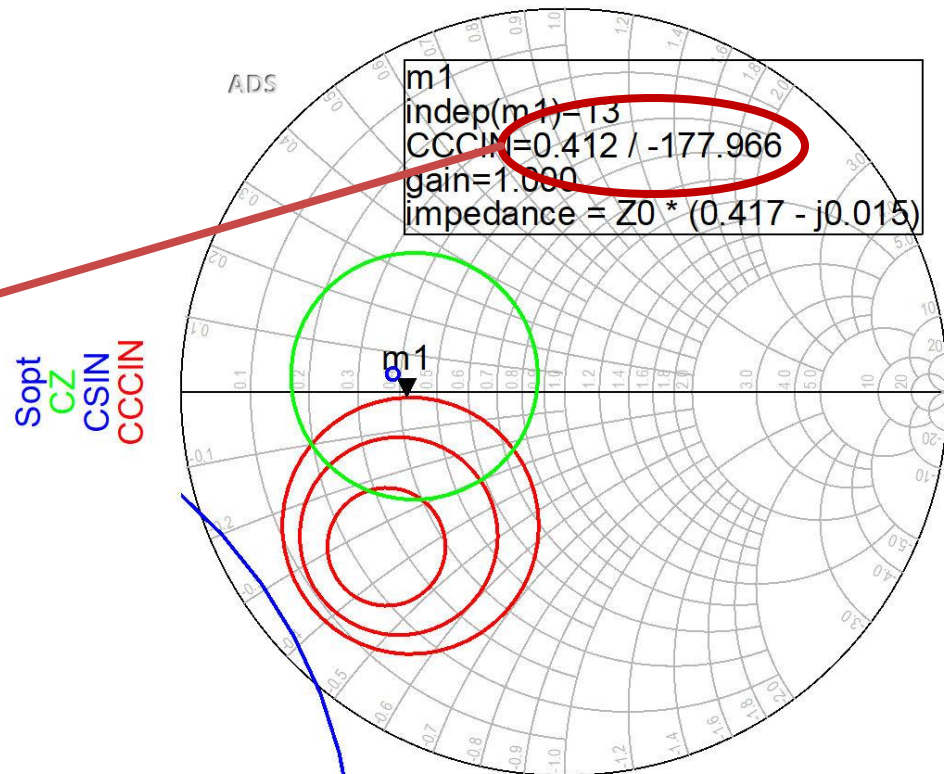
- Connecting the amplifier (transistor) directly to the source with  $Z_o$  generate a reflection coefficient seen towards the source equal with  $\mathbf{0}$  (complex,  $\Gamma_o = 0 + 0 \cdot j$ )
  - most of the time this reflection coefficient does not offer optimum noise/gain



# Matching – 2

- We plot on the complex plane (Smith Chart) the stability/gain/noise circles (depending on the particular application)
- We choose a point with a suitable position relative to these circles (also application dependent)
- We determine the input reflection coefficient corresponding to this point,  $\Gamma_S$

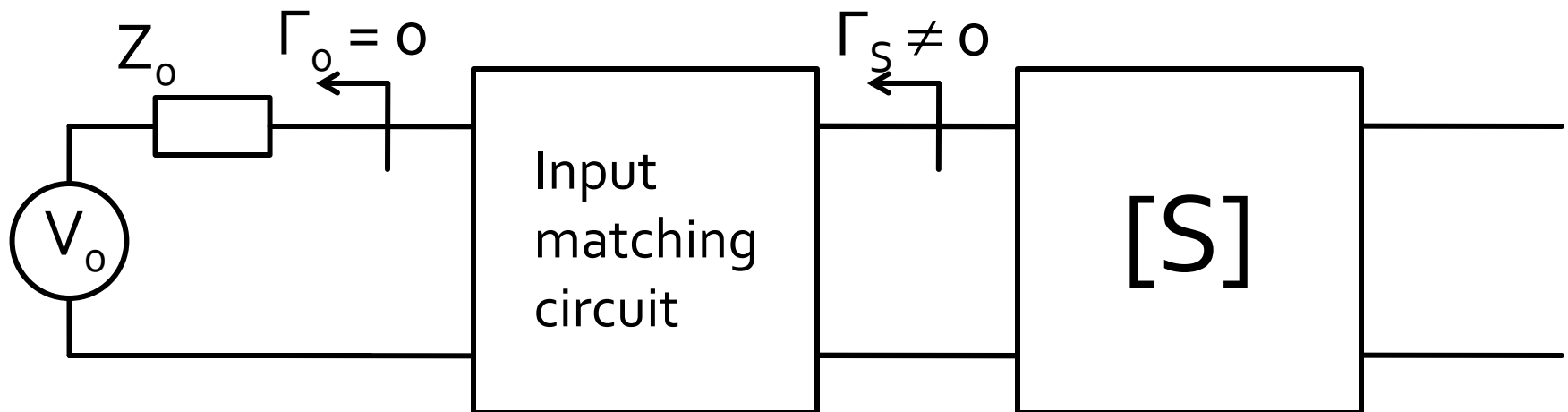
$$\Gamma_S = 0.412 \angle -177.966^\circ$$





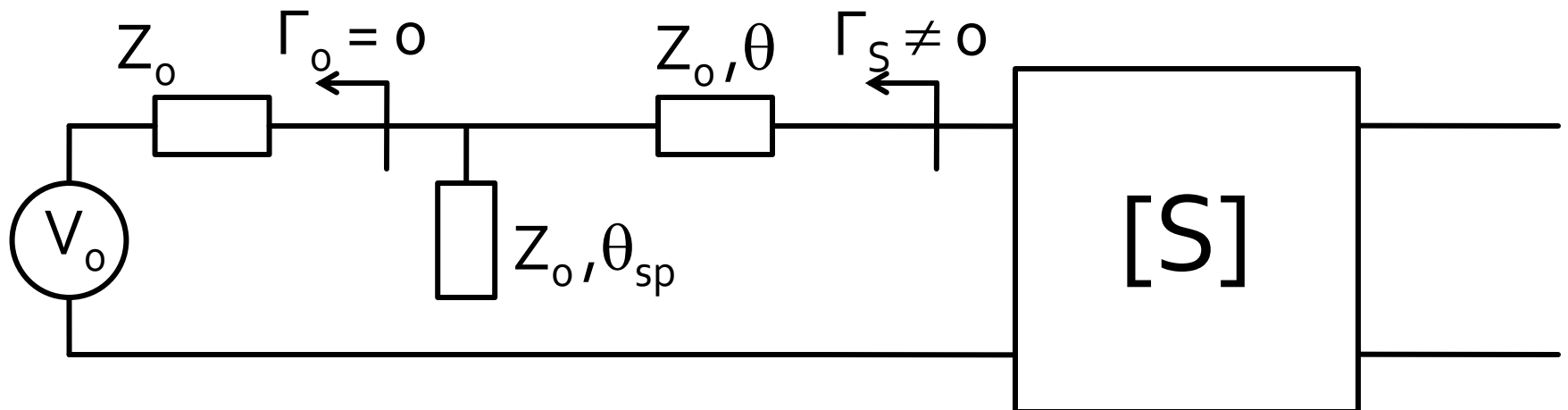
# Matching – 3

- We insert the input matching circuits which allows the transistor to see towards the source the previously determined reflection coefficient  $\Gamma_S$



# Matching – 4

- Easiest to design matching section consists in the insertion of (in order from the transistor towards the  $Z_0$  source):
  - a series  $Z_0$  line, with electrical length  $\theta$
  - a shunt stub, open-circuited, made from a  $Z_0$  line, with electrical length  $\theta_{sp}$

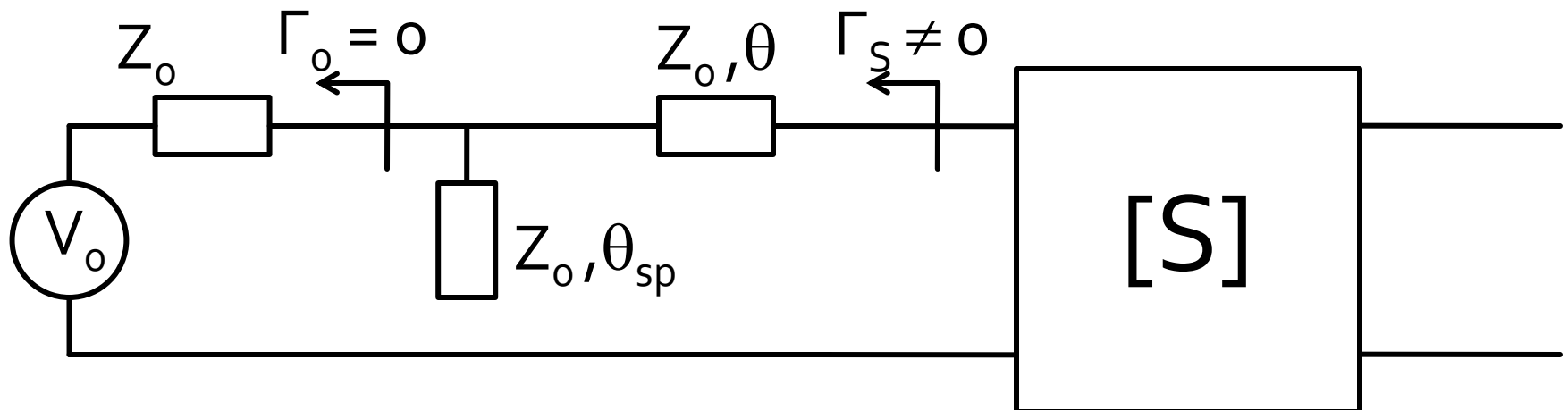


# Matching – 5

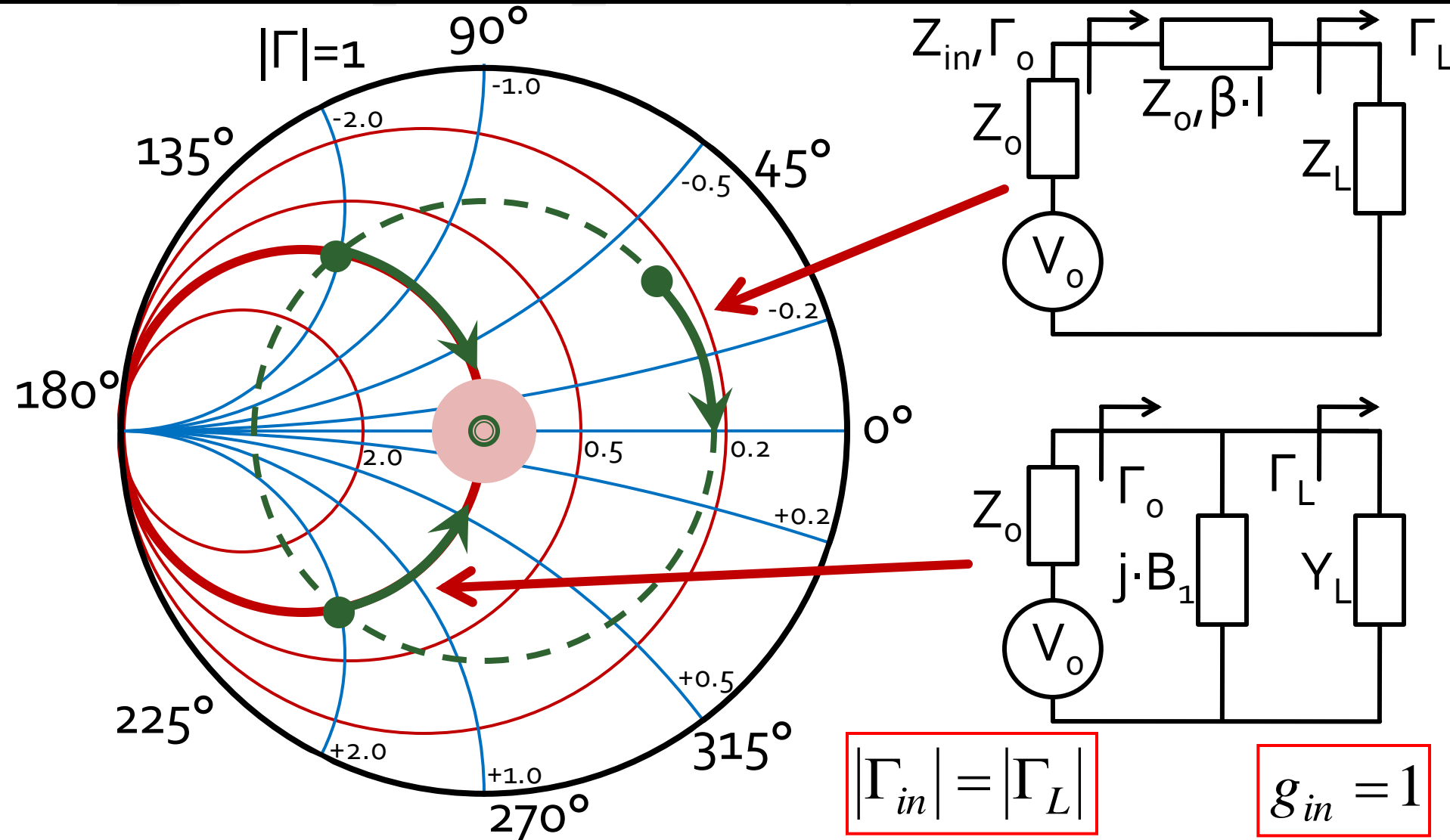
- Computation depends solely on  $\Gamma_S$  (magnitude and phase)

$$\cos(\varphi_S + 2\theta) = -|\Gamma_S| \quad \tan \theta_{sp} = \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

- The sign (+/-) chosen for the series line equation imposes the sign used for the shunt stub equation



# Shunt stub matching, C6-7



# Example, LNA @ 5 GHz

- ATF-34143 at  $V_{ds}=3V$   $I_d=20mA$ .
- @5GHz
  - $S_{11} = 0.64 \angle 139^\circ$
  - $S_{12} = 0.119 \angle -21^\circ$
  - $S_{21} = 3.165 \angle 16^\circ$
  - $S_{22} = 0.22 \angle 146^\circ$
  - $F_{min} = 0.54$  (tipic [dB])
  - $\Gamma_{opt} = 0.45 \angle 174^\circ$
  - $r_n = 0.03$

```
IATF-34143
IS-PARAMETERS at Vds=3V Id=20mA. LAST UPDATED 01-29-99

# ghz s ma r 50

2.0 0.75 -126 6.306 90 0.088 23 0.26 -120
2.5 0.72 -145 5.438 75 0.095 15 0.25 -140
3.0 0.69 -162 4.762 62 0.102 7 0.23 -156
4.0 0.65 166 3.806 38 0.111 -8 0.22 174
5.0 0.64 139 3.165 16 0.119 -21 0.22 146
6.0 0.65 114 2.706 -5 0.125 -35 0.23 118
7.0 0.66 89 2.326 -27 0.129 -49 0.25 91
8.0 0.69 67 2.017 -47 0.133 -62 0.29 67
9.0 0.72 48 1.758 -66 0.135 -75 0.34 46

!FREQ Fopt GAMMA OPT RN/Zo
!GHZ dB MAG ANG -
2.0 0.19 0.71 66 0.09
2.5 0.23 0.65 83 0.07
3.0 0.29 0.59 102 0.06
4.0 0.42 0.51 138 0.03
5.0 0.54 0.45 174 0.03
6.0 0.67 0.42 -151 0.05
7.0 0.79 0.42 -118 0.10
8.0 0.92 0.45 -88 0.18
9.0 1.04 0.51 -63 0.30
10.0 1.16 0.61 -43 0.46
```

# Example, LNA @ 5 GHz

- Low Noise Amplifier
- At the input matching a compromise is required between:
  - noise (~~input~~ constant noise figure circles)
  - gain (input constant gain circles)
  - stability (input stability circle)
- At the output matching noise **is not influenced**.  
A compromise is required between :
  - gain (output constant gain circles)
  - stability (output stability circle)

# Exemplu, LNA @ 5 GHz

$$U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{(1 - |S_{11}|^2) \cdot (1 - |S_{22}|^2)} = 0.094 \quad -0.783 \text{ dB} < G_T [\text{dB}] - G_{TU} [\text{dB}] < 0.861 \text{ dB}$$

$$G_{TU \text{ max}} = \frac{1}{1 - |S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |S_{22}|^2} = 17.83 \quad G_{TU \text{ max}} [\text{dB}] = 12.511 \text{ dB}$$

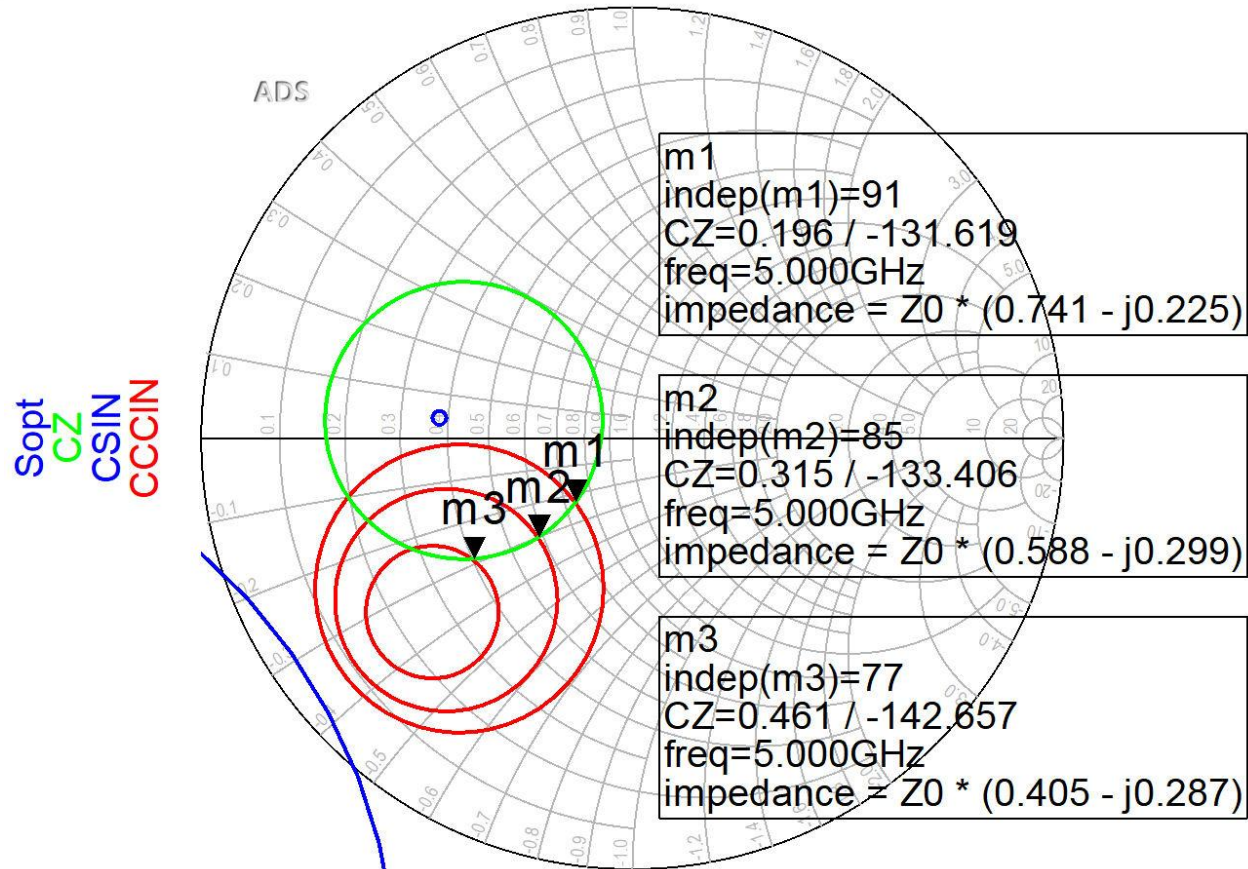
$$G_0 = |S_{21}|^2 = 10.017 = 10.007 \text{ dB}$$

$$G_{S \text{ max}} = \frac{1}{1 - |S_{11}|^2} = 1.694 = 2.289 \text{ dB}$$

$$G_{L \text{ max}} = \frac{1}{1 - |S_{22}|^2} = 1.051 = 0.215 \text{ dB}$$

- In this particular case  $G_{L \text{ max}} = 0.21 \text{ dB}$ , the transistor could be used directly connected to the  $50\Omega$  load
- The absence of the output matching circuit **is not** recommended. While the attainable power gain is low, its absence eliminates the possibility to use it to compensate an improper gain generated by the noise optimization of the input matching circuit

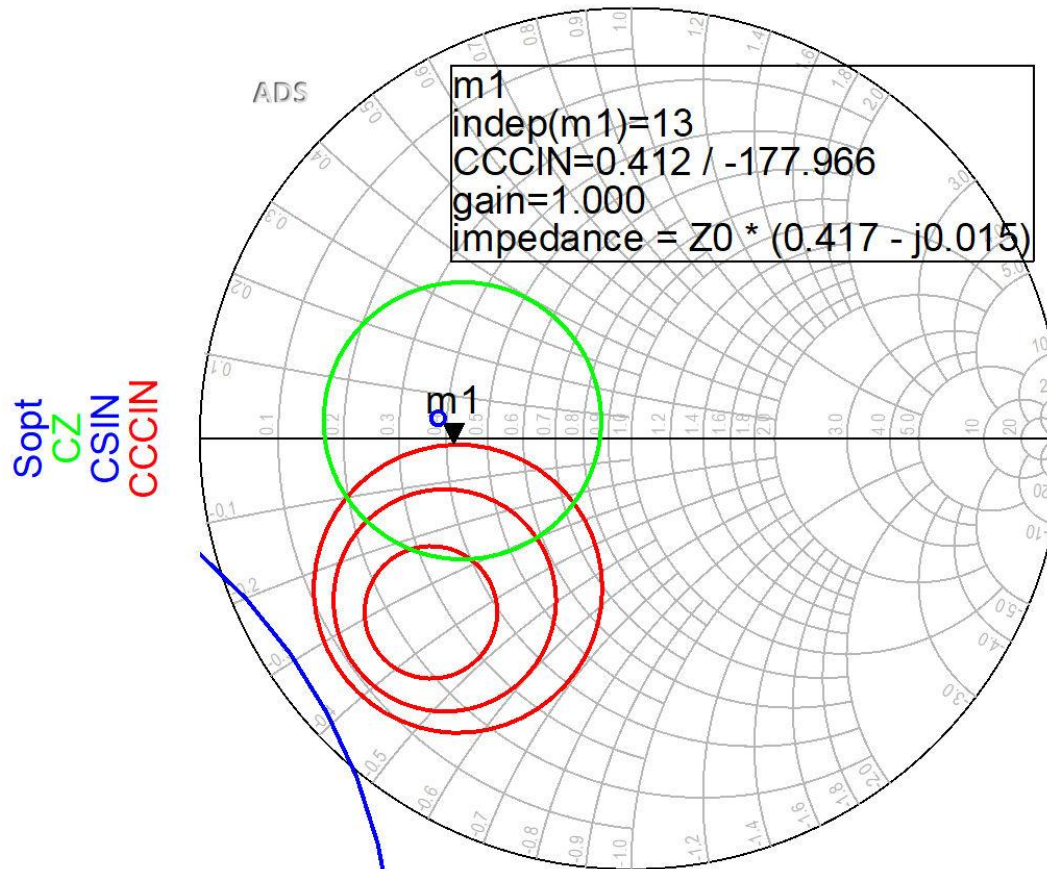
# Input matching circuit



- For the input matching circuit
  - noise circle CZ: 0.75dB
  - input constant gain circles CCCIN: 1dB, 1.5dB, 2 dB
- We choose (small Q → wide bandwidth) position m1



# Input matching circuit



- If we can afford a 1.2dB decrease of the input gain for better NF, Q ( $G_s = 1$  dB), position m1 above is better
- We obtain better (smaller) NF

# Input matching circuit

- Position  $m_1$  in complex plane (Smith Chart)

$$\Gamma_S = 0.412 \angle -178^\circ$$

$$|\Gamma_S| = 0.412; \quad \varphi = -178^\circ$$

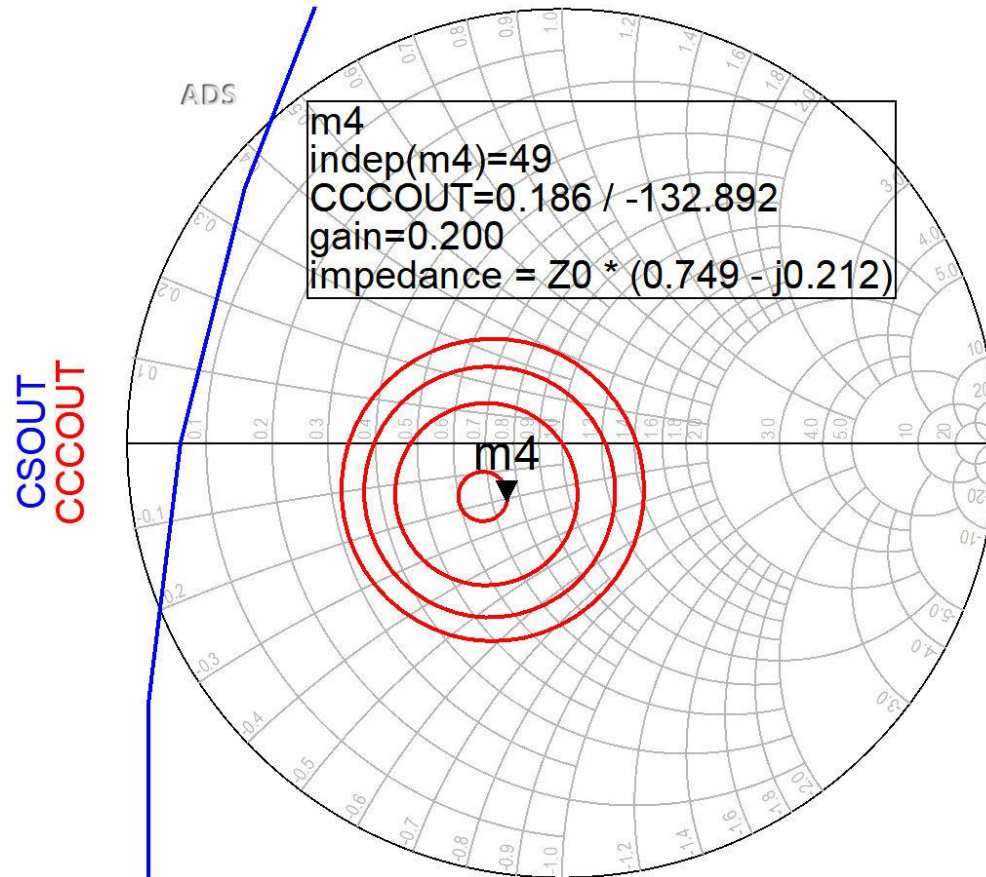
$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\text{Im}[y_S(\theta)] = \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\cos(\varphi + 2\theta) = -0.412 \Rightarrow (\varphi + 2\theta) = \pm 114.33^\circ$$

$$(\varphi + 2\theta) = \begin{cases} +114.33^\circ \\ -114.33^\circ \end{cases} \quad \theta = \begin{cases} 146.2^\circ \\ 31.8^\circ \end{cases} \quad \text{Im}[y_S(\theta)] = \begin{cases} -0.904 \\ +0.904 \end{cases} \quad \theta_{sp} = \begin{cases} 137.9^\circ \\ 42.1^\circ \end{cases}$$

# Output matching circuit



- output constant gain circles CCCOUT: -0.4dB, -0.2dB, 0dB, +0.2dB
- The lack of noise restrictions allows optimization for better gain (close to maximum – position m4)

# Output matching circuit

- Position  $m_4$  in complex plane (Smith Chart)

$$\Gamma_L = 0.186 \angle -132.9^\circ$$

$$|\Gamma_L| = 0.186; \quad \varphi = -132.9^\circ$$

$$\cos(\varphi + 2\theta) = -|\Gamma_L|$$

$$\text{Im}[y_L(\theta)] = \frac{-2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}} = -0.379$$

$$\cos(\varphi + 2\theta) = -0.186 \Rightarrow (\varphi + 2\theta) = \pm 100.72^\circ$$

$$(\varphi + 2\theta) = \begin{cases} +100.72^\circ \\ -100.72^\circ \end{cases} \quad \theta = \begin{cases} 116.8^\circ \\ 16.1^\circ \end{cases} \quad \text{Im}[y_L(\theta)] = \begin{cases} -0.379 \\ +0.379 \end{cases} \quad \theta_{sp} = \begin{cases} 159.3^\circ \\ 20.7^\circ \end{cases}$$

# LNA

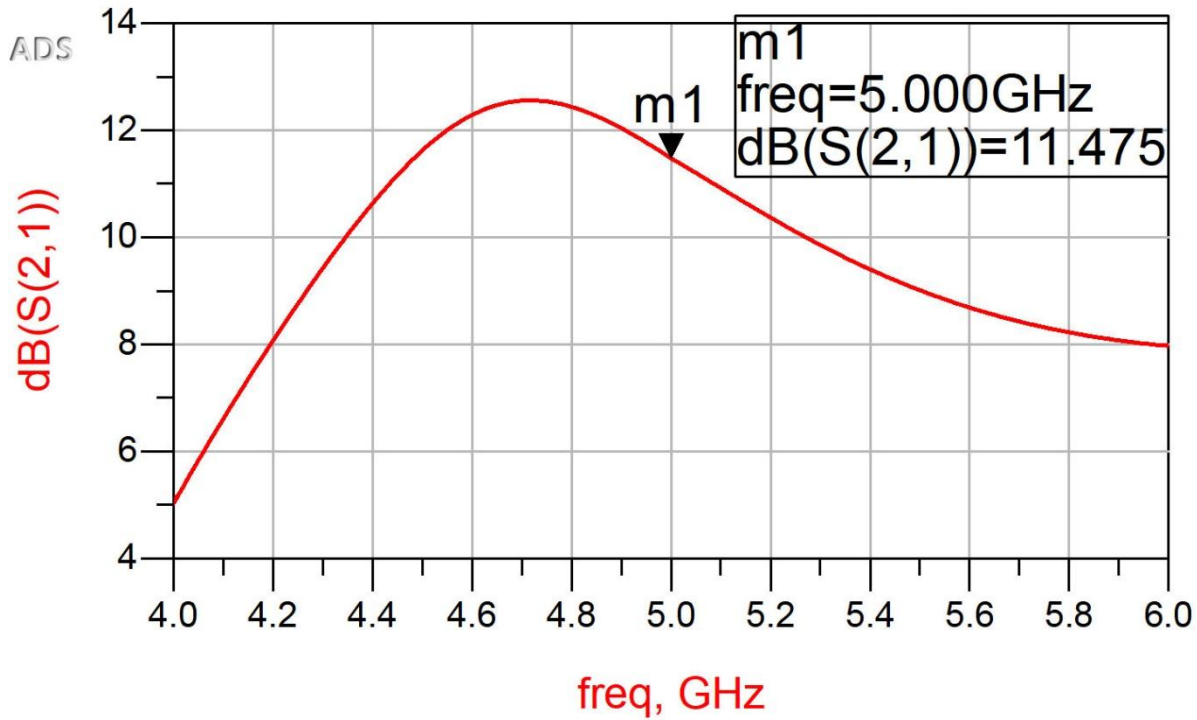
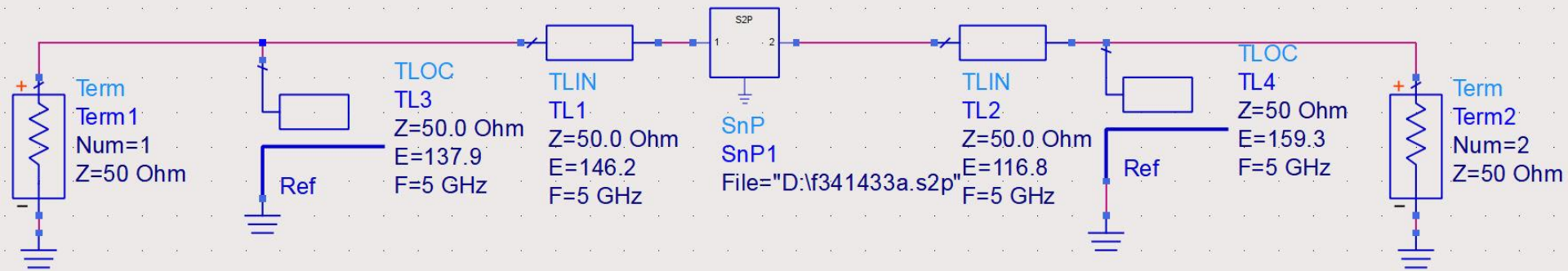
- We estimate a gain (in unilateral assumption,  $\pm 0.9$  dB)

$$G_T [dB] = G_S [dB] + G_0 [dB] + G_L [dB]$$

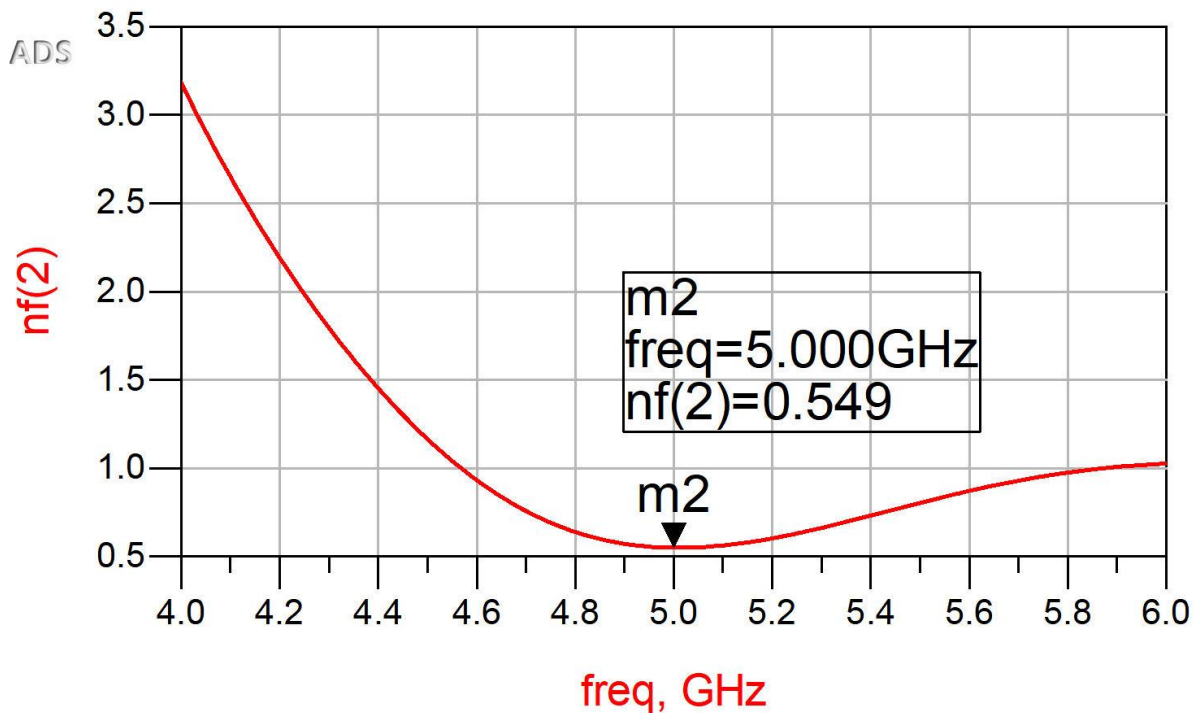
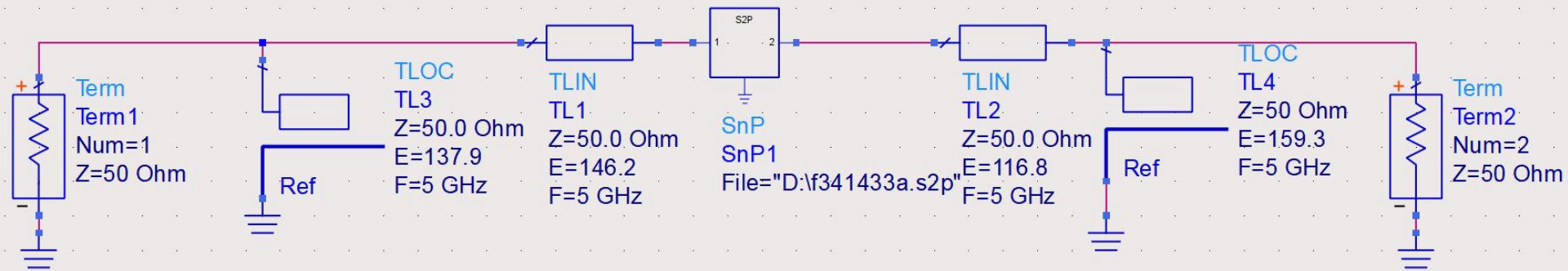
$$G_T [dB] = 1 \text{ dB} + 10 \text{ dB} + 0.2 \text{ dB} = 11.2 \text{ dB}$$

- We estimate a noise factor well below 0.75dB (quite close to the minimum  $\sim 0.6$  dB)

# ADS



# ADS



# Contact

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- [rdamian@etti.tuiasi.ro](mailto:rdamian@etti.tuiasi.ro)